

آموزش حسابان یازدهم

محاسبه حد توابع مثلثاتی

(فصل پنجم – درس چهارم)

علی جبرا | سایت تخصصی آموزش ریاضی

ALIGEBRA.COM

۰۹۱۲۷۷۴۴۳۸۹ – ۰۹۱۲۷۷۴۴۲۸۱

کلیه حقوق مادی و معنوی این اثر متعلق به سایت Algebra.com است و هرگونه استفاده از این اثر و انتشار آن در پایگاه های مجازی بدون کسب مجوز ممنوع است و متخلفان تحت پیگرد قانونی قرار می گیرند.

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$1 + \cot^2 x = \frac{1}{\sin^2 x}$$

$$\sin 2x = 2 \sin x \cdot \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x =$$

$$2 \cos^2 x - 1$$

$$1 - 2 \sin^2 x$$

$$\frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$1 + \cos 2x = 2 \cos^2 x \checkmark \checkmark$$

$$1 - \cos 2x = 2 \sin^2 x \checkmark \checkmark$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$1 + \cos \frac{x}{2} = 2 \cos^2 \frac{x}{4}$$

$$1 - \cos \frac{x}{2} = 2 \sin^2 \frac{x}{4}$$

$$\frac{1 + \cos x}{2} = \cos^2 \frac{x}{2}$$

$$1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$1 + \cos 2x = 2 \cos^2 x$$

$$1 - \cos 2x = 2 \sin^2 x$$

$$1) \sin ax \rightarrow a \cos ax$$

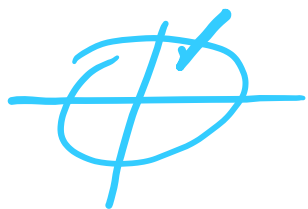
$$2) \cos ax \rightarrow -a \sin ax$$

$$3) \tan ax \rightarrow a(1 + \tan^2 ax)$$

$$4) \cot ax \rightarrow -a(1 + \cot^2 ax)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\cos x} = \frac{0}{0} \xrightarrow{\text{HoP}} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{-\sin x} = \frac{0}{-1} = 0 \quad \checkmark$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{0}{0} \xrightarrow{\text{HoP}} \lim_{x \rightarrow 0} \frac{0 + \sin x}{2x} = \frac{0}{0} \xrightarrow{\text{HoP}} \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2} \quad \checkmark$$



حاصل $\lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos x}}{\tan 2x}$ کدام است؟ ۱

$$\lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}}}{\tan 2x} = \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}}}{\tan 2x} = \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}}}{\tan 2x}$$

$$\frac{0}{0} \xrightarrow{\text{AOP}} \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}} \left(-\frac{1}{2} \sin \frac{x}{2} \right)}{2(1 + \tan^2 2x)} = \frac{\sqrt{1 + \cos \frac{\pi}{2}} \left(-\frac{1}{2} \right)}{2} = -\frac{\sqrt{1}}{2}$$

$$\lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}}}{\frac{\sin 2x}{\cos 2x}} = \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}} \cos 2x}{\sin 2x} = \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \cos \frac{x}{2}} \cos 2x}{\sin 2x} = \frac{\sqrt{1 + \cos \frac{\pi}{2}} \cos 2\pi}{\sin 2\pi} = \frac{\sqrt{1} \cdot 1}{0}$$

سایت علی جبرا Aligebra.com

پشتیبانی ۰۹۱۲۷۷۶۶۲۸۱ - ۰۹۱۲۷۷۶۶۳۸۹

$\left| \cancel{2} \sin x \cos x - \cancel{2} \cos x \right| = \left| \cancel{2}^{\oplus} \cos x \left(\sin x - \cancel{1}^{\ominus} \right) \right|$
 کدام است؟ $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \frac{\cos^3 x}{|\sin 2x - 2 \cos x|}$ حاصل ۲

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\cos^2 x \cdot \cos x}{\cancel{2} \cos x (1 - \sin x)} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{(1 - \sin^2 x) \cdot \cos x}{\cancel{2} \cos x (1 - \sin x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\cancel{(1 - \sin x)} (1 + \sin x) \cdot \cancel{\cos x}}{\cancel{2} \cancel{\cos x} \cancel{(1 - \sin x)}} = \frac{1+1}{2} = 1$$

حد عبارت $\frac{\sqrt{\tan x} - \sqrt{\frac{1}{\tan x}}}{\cos 2x}$ وقتی $x \rightarrow \frac{\pi}{4}$ کدام است؟

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{\tan x} - \frac{1}{\sqrt{\tan x}}}{\cos 2x}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\tan x - 1}{\sqrt{\tan x}}}{\frac{1 - \tan^2 x}{1 + \tan^2 x}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\tan x - 1)(1 + \tan^2 x)}{\sqrt{\tan x} (1 - \tan x)(1 + \tan x)} = \frac{-1}{1} = -1$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{\cos 2x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\sin x}{\cos x} - 1}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos x (\cos x - \sin x) (\cos x + \sin x)} \\ &= \frac{-1}{\sqrt{\frac{\pi}{4}} (\sqrt{1})} = \frac{-1}{1} = -1 \end{aligned}$$

۴ حاصل $\lim_{x \rightarrow \frac{3\pi}{4}} \frac{\tan^2 x - 1}{\cos^2 x}$ کدام است؟

$$\lim_{x \rightarrow \frac{3\pi}{4}} \frac{\cancel{\tan^2 x} - 1}{\cancel{1 - \tan^2 x}} = \lim_{x \rightarrow \frac{3\pi}{4}} -(1 + \tan^2 x) = -2$$

$$\begin{aligned} \lim_{x \rightarrow \frac{3\pi}{4}} \frac{\frac{\sin^2 x}{\cos^2 x} - 1}{\cos^2 x - \sin^2 x} &= \lim_{x \rightarrow \frac{3\pi}{4}} \frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{3\pi}{4}} \frac{\cancel{\sin^2 x - \cos^2 x}}{\cos^2 x (\cancel{\cos^2 x - \sin^2 x})} \\ &= \frac{-1}{1} = -1 \end{aligned}$$

مقدار $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{\tan^3 x}$ کدام است؟ ۵

$$\lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{2 \sin x (1 - \cos x)}{\sin x \cdot \sin^2 x} = \lim_{x \rightarrow 0} \frac{\cancel{2 \sin x} (1 - \cancel{\cos x})}{\cancel{\sin x} (1 - \cancel{\cos x}) (1 + \cos x)}$$

$\frac{1}{2} \leftarrow \cos^2 x$
 $1 - \cos^2 x \leftarrow$

$$= \lim_{x \rightarrow 0} \frac{2}{1 + \cos x} = \frac{2}{1 + 1} = 1$$

حاصل $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{\sin^2 x}$ کدام است؟ ۶

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{1 - \cos^2 x} \times \frac{1 + \sqrt{\cos x}}{1 + \sqrt{\cos x}} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 - \cos x)(1 + \cos x)(1 + \sqrt{\cos x})} = \frac{1}{4}$$

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{(1 - \cos x)(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{(1 - \sqrt{\cos x})(1 + \sqrt{\cos x})(1 + \cos x)} = \frac{1}{4}$$

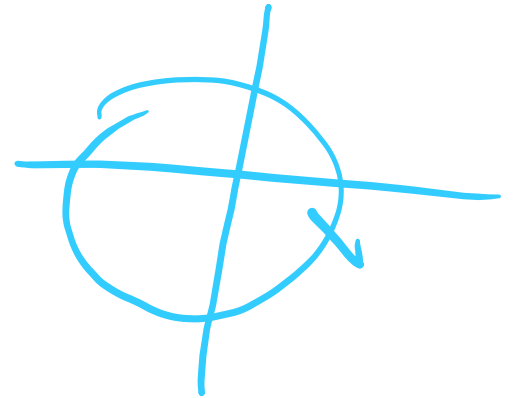
حاصل $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{\cos 2x}$ کدام است؟ ۷

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{\frac{1 - \tan^2 x}{1 + \tan^2 x}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cancel{(\tan x - 1)} (1 + \tan^2 x)}{\cancel{(1 - \tan x)} (1 + \tan x)} = -1$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\sin x}{\cos x} - 1}{\cos^2 x - \sin^2 x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\sin x - \cos x}{\cos x}}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cancel{\sin x - \cos x}}{\cos x (\cancel{\cos x - \sin x}) (\cos x + \sin x)} \\ &= \frac{-1}{\frac{\sqrt{2}}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right)} = -1 \end{aligned}$$

حد عبارت $\frac{\tan^2 x \sqrt{1 - \cos^2 x}}{\sin^2 x}$ وقتی $x \rightarrow 0^-$ کدام است؟ (۸)

$$\lim_{x \rightarrow 0^-} \frac{\frac{\sin^2 x}{\cos^2 x} \cdot \sqrt{1 - \cos^2 x}}{\sin^2 x} = \lim_{x \rightarrow 0^-} \frac{\sin^2 x \cdot \sqrt{1 - \cos^2 x}}{\sin^2 x \cdot \cos^2 x}$$



$$= \lim_{x \rightarrow 0^-} \frac{-\sqrt{1} \cdot \cancel{\sin x} \cos x \cdot \cancel{\sin x}}{\cancel{\sin x} \cdot \cancel{\sin x} \cdot \cos^2 x} = -\frac{1}{1}$$

۹ حاصل $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \cos 2x - 1}{2 \sin^2 x + \sin x - 1}$ کدام است؟

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2(1 - 2 \sin^2 x) - 1}{2 \sin^2 x + 2 \sin x - \sin x - 1} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 4 \sin^2 x}{2 \sin x (\sin x + 1) - (\sin x + 1)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(1 - 2 \sin x)(1 + 2 \sin x)}{(\sin x + 1)(2 \sin x - 1)} = \frac{-2}{\frac{\pi}{6}} = \frac{-4}{\pi}$$

$$[x] + [-x] : \begin{cases} x \in \mathbb{Z} \rightarrow 0 \\ x \notin \mathbb{Z} \rightarrow -1 \end{cases}$$

کدام است؟ $\lim_{x \rightarrow \frac{3\pi}{2}} \underbrace{([x] + [-x])}_{-1} \frac{\cos^2 x}{1 + \sin^3 x}$ حاصل ۱۰

$$\begin{aligned} \lim_{x \rightarrow \frac{3\pi}{2}} \frac{-(1 - \sin^2 x)}{1 + \sin^3 x} &= \lim_{x \rightarrow \frac{3\pi}{2}} \frac{-(1 - \sin x)(1 + \sin x)}{(1 + \sin x)(1 - \sin x + \sin^2 x)} \\ &= \frac{-(1 + 1)}{1 + 1 + 1} = \frac{-2}{3} \end{aligned}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$