

آموزش ریاضی

انتگرال نامعین مثلثاتی

[Algebra.com](http://Algebra.com)

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln |\cos ax|$$

$$\int \cot ax \, dx = \frac{1}{a} \ln |\sin ax|$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{Arctan} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{ArcSin} \frac{x}{a}$$

$$\int \tan^2 x \, dx = \tan x - x$$

$$\int \cot^2 x \, dx = -\cot x - x$$

۱- حاصل  $\int \frac{\tan^{\omega} x + \cot^{\mu} x}{\cos^{\omega} x} dx$  کدام است؟

$$\begin{aligned} u &= \tan x \\ du &= \frac{1}{\cos^2 x} dx \end{aligned}$$

$$\cot^{\mu} x = \frac{1}{\tan^{\mu} x} = \frac{1}{u^{\mu}} = u^{-\mu}$$

$$\int (u^{\omega} + u^{-\mu}) du = \frac{u^{\omega+1}}{\omega+1} + \frac{u^{-\mu+1}}{-\mu+1} = \frac{\tan^{\omega+1} x}{\omega+1} - \frac{\tan^{-\mu+1} x}{\mu-1}$$

$$= \frac{\tan^{\omega+1} x}{\omega+1} - \frac{\cot^{\mu-1} x}{\mu-1}$$

۱-۲ اگر  $\int (2\sin^2 x + \cos^2 x) dx = g(x)$  باشد، حاصل  $\int (2\cos^2 x + \sin^2 x) dx$  چقدر است؟

$$g = \int (2\sin^2 x + 1 - \sin^2 x) dx = \int (1 + \sin^2 x) dx$$

$$\int (2\cos^2 x + 1 - \cos^2 x) dx = \int (\cos^2 x + 1) dx = ?$$

$$\int (\underline{1+1} - \cos^2 x) dx = \int (\underline{-\cos^2 x - 1 + 3}) dx =$$

$$= \ominus \int (1 + \cos^2 x) dx + 3x = g \rightarrow \int (1 + \cos^2 x) dx = 3x - g$$

۳- حاصل  $\int \frac{\sin^3 x + \cos^3 x}{\cos^5 x} dx$  کدام است؟

$$\int \frac{1}{\cos^5 x} \left( \frac{\sin^3 x}{\cos^3 x} + \frac{\cos^3 x}{\cos^3 x} \right) dx = \int \frac{1}{\cos^5 x} (\tan^3 x + 1) dx$$

$$\begin{aligned} | u &= \tan x \\ | du &= \frac{1}{\cos^2 x} dx \end{aligned}$$

$$\rightarrow \int (u^3 + 1) du = \frac{u^4}{4} + u$$

$$\therefore \text{جواب} = \frac{1}{4} \tan^4 x + \tan x$$

۴- با فرض آن که  $\int \frac{dx}{1 - \cos^2 2x} = \frac{1 - f(x)}{\sin 2x} + c$  ، ضابطه ی  $f(x)$  کدام می تواند باشد؟

$$\int \frac{dx}{\sin^2 2x} = \int (1 + \cot^2 2x) dx = \frac{-1}{2} \cot 2x$$

$$\frac{-1}{2} \cdot \frac{\cos 2x}{\sin 2x} \longrightarrow 1 - f(x) = \frac{-1}{2} \cos 2x$$

$$f(x) = 1 + \frac{1}{2} \cos 2x = 1 + \frac{1}{2} (2 \cos^2 x - 1) = \cos^2 x + \frac{1}{2}$$

۵- اگر  $\int \cos \underline{2x} \sin \underline{4x} dx = \underline{m} \cos \underline{6x} - \frac{1}{\underline{4}} \cos \underline{nx} + c$  ، مقدار  $mn$  چه عددی است؟

$$\underline{\sin a} \cdot \underline{\cos b} = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\int \underline{\cos 2x} \underline{\sin 4x} dx = \frac{1}{2} \int (\sin 4x + \sin 2x) dx$$

$$= \frac{1}{2} \left( \frac{-1}{4} \cos 4x - \frac{1}{2} \cos 2x \right) = \underline{\frac{-1}{8}} \cos 4x - \frac{1}{4} \underline{\cos 2x}$$

$$m = \frac{-1}{8}$$

$$n = 2$$

$$mn = \frac{-1}{4}$$

6- حاصل  $\int (\sin x \cos^5 x - \cos x \sin^5 x) dx$  کدام است؟  $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$$\int \sin x \cos x (\cos^2 x - \sin^2 x) dx = \int \frac{1}{4} \sin 2x \cdot \cos 2x dx$$

$$= \int \frac{1}{4} \cdot \frac{1}{4} \sin 2x dx = \frac{1}{16} \int \sin 2x dx = -\frac{1}{16} \cos 2x$$

$$u = \cos x \rightarrow \int -u^4 du = -\frac{u^5}{5} = -\frac{\cos^5 x}{5}$$

$$du = -\sin x dx$$

$$u = \sin x \rightarrow \int -u^4 du = -\frac{u^5}{5} = -\frac{\sin^5 x}{5}$$

$$du = \cos x dx$$

$$\rightarrow \frac{-\cos^5 x}{5} - \frac{\sin^5 x}{5}$$



۷- حاصل انتگرال  $\int \frac{1 + \sin^2 x}{\cos^2 x} dx$  کدام است؟

$$\int \left( \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \right) dx = \int \left( 1 + \underline{\underline{\tan^2 x}} + \underline{\underline{\tan^2 x}} \right) dx$$

$$= \tan x + \tan x - x = 2 \tan x - x$$

۸- حاصل  $\int \frac{\cos^3 x}{\cos x} dx$  کدام است؟

$$\int \frac{\cos^3 x - \cos x}{\cos x} dx = \int (\underline{\cos^2 x} - 1) dx \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= \int (1 + \cos 2x - 1) dx = \sin 2x - x$$

$$\underline{\underline{\sin^3 x = 3 \sin x - 4 \sin^3 x}}$$

$$\underline{\underline{\sin^2 x = \frac{1 - \cos 2x}{2}}}$$

۹- حاصل  $\int \frac{1+2x}{1+x^2} dx$  کدام است؟

$$\int \frac{1}{1+x^2} dx + \int \frac{2x}{1+x^2} dx$$

$$= \text{Arctan } x + \ln(1+x^2)$$

$$u = 1+x^2 \rightarrow \int \frac{2x}{1+x^2} dx = \int \frac{du}{u} = \ln u = \ln |1+x^2|$$

۱۰- حاصل  $\int (\sin x + 2)(x \sin x + x^2 \cos x + 2x) dx$  برابر است با:

$$\int (\sin x + 2) \cdot \underline{x} (\sin x + x \cos x + 2) dx$$

$$= \int \underbrace{(x \sin x + 2x)}_u \underbrace{(\sin x + x \cos x + 2)}_{du} dx$$

$$u = x \sin x + 2x \quad \Rightarrow \quad du = (\sin x + x \cos x + 2) dx$$

$$\int u' du = \frac{u^r}{r} = \frac{1}{r} (x \sin x + 2x)^r$$

۱۱- اگر  $\int (x \cos x \sqrt{\sin x} - \sin x \sqrt{\sin x})^n dx = \frac{(x \cos x - \sin x)^m}{n} + c$  باشد آنگاه:

$m + n = -6$  (۴)

$m + n = 4$  (۳)

$m + n = 0$  (۲)

$m + n = 6$  (۱)

$$\int x (\sqrt{\sin x} (x \cos x - \sin x))^n dx = \int \underline{x \sin x} (\underline{x \cos x - \sin x})^n dx$$

$U = x \cos x - \sin x \rightarrow dU = (\cos x - x \sin x - \cos x) dx = \underline{-x \sin x} dx$

$\int U^n dU = \frac{U^{n+1}}{n+1} = \frac{-1}{n+1} (x \cos x - \sin x)^{n+1} \rightarrow m = n+1$

$n = -3$

$\frac{-1}{n+1} = \frac{-1}{-3+1} = \frac{-1}{-2} = \frac{1}{2}$

۱۲- اگر  $\int \sin x \left( 1 + \frac{1}{\cos^2 x} \right) dx = \frac{f(x)}{\cos x} + c$  و  $x \neq \frac{k\pi}{2}$  آن گاه تابع  $f(x)$  کدام است؟

$$\int \left( \sin x + \frac{\sin x}{\cos^2 x} \right) dx = -\cos x + \frac{1}{\cos x}$$

$$u = \cos x \rightarrow \int \frac{du}{u^2} = - \int u^{-2} du = - \left( \frac{u^{-1}}{-1} \right) = \frac{1}{u} = \frac{1}{\cos x}$$

$du = -\sin x dx$

$$\therefore \text{جواب} = \frac{-\cos^2 x + 1}{\cos x} = \frac{\sin^2 x}{\cos x} \rightarrow f(x) = \sin^2 x$$

۱۳- اگر  $f''(x) = \sin \frac{x}{2}$  باشد،  $f(x)$  کدام است؟

$$f'(x) = \int \sin \frac{x}{2} dx = -2 \cos \frac{x}{2} + C$$

$$f(x) = \int (-2 \cos \frac{x}{2} + C) dx = -4 \sin \frac{x}{2} + Cx + d$$

۱۴- اگر  $F(x) = \int \frac{2x + \sqrt{1+x^2}}{\sqrt{1-x^2}} dx$ ، با شرط  $F(0) = 0$  مقدار  $F(\frac{\sqrt{2}}{2})$  کدام است؟

$$\int \left( \frac{2x}{\sqrt{1-x^2}} + \frac{\sqrt{1+x^2}}{\sqrt{(1-x^2)(1+x^2)}} \right) dx = \text{ArcSin } x^2 + \text{ArcSin } x + C$$

$f(0) = 0 \rightarrow 0 + 0 + C = 0 \rightarrow C = 0$

$u = x^2 \rightarrow du = 2x dx$

$$\int \frac{du}{\sqrt{1-u^2}} = \text{ArcSin } u = \text{ArcSin } x^2$$

$$f\left(\frac{\sqrt{2}}{2}\right) = \text{ArcSin}\left(\frac{1}{2}\right) + \text{ArcSin}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$$



۱۵- اگر  $F(x) = \int \frac{x^r + x + 2}{1+x^r} dx$ ، با شرط  $F(0) = 0$ ، مقدار  $F(1)$  کدام است؟

$$\int \left( \frac{x(x^{r+1})}{1+x^r} + \frac{2}{1+x^r} \right) dx = \int \left( x + \frac{2}{1+x^r} \right) dx$$

$$= \frac{x^r}{r} + 2 \operatorname{Arctan} x + C$$

$F(0)=0 \rightarrow 0+0+C=0 \rightarrow C=0$

$$F(1) = \frac{1}{r} + 2 \operatorname{Arctan}(1) = \frac{1}{r} + \frac{\sqrt{2}}{r} = \frac{1}{r} (\sqrt{2}+1)$$

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