

آموزش ریاضی

انتگرال نامعین

Algebra.com

$$\int c \, dx = cx$$

$$\int \omega \, dx = \omega x$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1}$$

$$\int x^a \, dx = \frac{x^4}{4}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{\omega}{x} \, dx = \omega \ln x$$

$$x^{-1} = \frac{x^0}{0} \int$$

۱-۱ اگر $\int \sqrt{\left(x^\mu - \frac{1}{x^\mu}\right)^2 + 4} dx = \frac{f(x)}{3x} + c$ آنگاه $f(x)$ کدام است؟

$$\int \sqrt{x^\mu + \frac{1}{x^\mu} - \cancel{\frac{2}{x^\mu}} + 4} dx = \int \sqrt{\left(x^\mu + \frac{1}{x^\mu}\right)^2} dx$$

$$= \int \left(x^\mu + \frac{1}{x^\mu}\right) dx = \int \left(\underline{x^\mu} + \underline{x^{-\mu}}\right) dx = \frac{x^\mu}{\mu} + \frac{x^{-1}}{-1}$$

$$\frac{x^\mu}{\mu} - \frac{1}{x} = \frac{x^\mu - \mu}{\mu x} = \frac{f(x)}{\mu x}$$

$$\rightarrow f(x) = x^\mu - \mu$$

$$u = x - 1 \rightarrow \begin{cases} x = u + 1 \\ dx = du \end{cases}$$

۱-۲ اگر $\int \underline{x} \sqrt{\underline{x-1}} dx = \underline{\sqrt{x-1}} f(x) + c$ باشد $f(x)$ کدام است؟

$$\int (u+1) \sqrt{u} du = \int (u^{3/2} + u^{1/2}) du = \frac{1u^{5/2}}{5/2} + \frac{1u^{3/2}}{3/2}$$

$$\frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} = u^{1/2} \left(\frac{2}{5} u^2 + \frac{2}{3} u \right) = \sqrt{x-1} \left(\frac{2}{5} (x-1)^2 + \frac{2}{3} (x-1) \right)$$

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad \checkmark$$

۳- اگر $\int \frac{x^2 - 2x + 2}{\sqrt{x-1}} dx = \frac{\sqrt{x-1}}{5} f(x) + c$ مقدار $f(x)$ چقدر است؟

$$\int \frac{x^2 - 2x + 1 + 1}{\sqrt{x-1}} dx = \int \frac{(x-1)^2 + 1}{\sqrt{x-1}} dx$$

$U = x-1$
 $x = U+1$
 $du = dx$

$$\int \frac{U^2 + 1}{\sqrt{U}} dU = \int (U^2 + 1) U^{-1/2} dU = \int (U^{5/2} + U^{-1/2}) dU = \frac{U^{5/2}}{5/2} + \frac{U^{1/2}}{1/2}$$

$$= \frac{2}{5} U^{5/2} + \frac{2}{1} U^{1/2} = \frac{2}{5} U^{5/2} + 2 U^{1/2} = \frac{2}{5} (U^2 + 1) \sqrt{U} = \frac{\sqrt{x-1}}{5} (x^2 - 2x + 2 + 1)$$

$$f = x^2 - 2x + 2$$

۴-اگر $\int \sqrt{\frac{1+\sqrt{x}}{x}} dx = \frac{(1+\sqrt{x})^m}{n} + c$ حاصل $m+n$ کدام است؟

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$$

$$U = 1 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx \rightarrow \frac{dx}{\sqrt{x}} = 2du$$

$$\int \sqrt{u} \cdot 2du = 2 \int u^{\frac{1}{2}} du = 2 \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{(1+\sqrt{x})^{\frac{3}{2}}}{\frac{3}{2}}$$

$$m = \frac{3}{2}$$

$$n = \frac{3}{2}$$

$$m+n = 3$$

۵- حاصل $\int (x\sqrt{x} - 1)^2 dx$ کدام است؟

$$\int (x^{\frac{3}{2}} + 1 - 2x^{\frac{3}{2}}) dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x - 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= \frac{2}{3} x^{\frac{3}{2}} + x - \frac{4}{3} x^{\frac{3}{2}}$$

۶-اگر $\int \frac{5x^2 + 6x}{2\sqrt{x}} dx = f(x) \cdot \sqrt{x} + c$ ، آن گاه $f(x)$ کدام است؟

$$\int \frac{5x^2}{2\sqrt{x}} dx + \int \frac{6x}{2\sqrt{x}} dx = \frac{5}{2} \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx$$

$$= \frac{5}{2} \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = x + 2x^{\frac{3}{2}}$$

$$x^{\frac{1}{2}} (x^2 + 2x) = \sqrt{x} (x^2 + 2x)$$

$$f(x) = x^2 + 2x$$

۷- حاصل انتگرال $\int x(\sqrt{x+1})dx$ کدام است؟ $U = x+1 \rightarrow x = U-1 \rightarrow dx = dU$

$$\int (U-1)\sqrt{U} dU = \int (U^{\frac{3}{2}} - U^{\frac{1}{2}}) dU = \frac{U^{\frac{5}{2}}}{\frac{5}{2}} - \frac{U^{\frac{3}{2}}}{\frac{3}{2}}$$

$$\frac{2}{5} U^{\frac{5}{2}} - \frac{2}{3} U^{\frac{3}{2}} = \frac{2}{5} \sqrt{U^5} - \frac{2}{3} \sqrt{U^3}$$

$$= \frac{2}{5} \sqrt{(x+1)^5} - \frac{2}{3} \sqrt{(x+1)^3}$$

۸- اگر $F(x)$ تابع اولیه $f(x) = (6x - 1)\sqrt{3x - 1}$ باشد، $F(\frac{2}{3}) - F(\frac{1}{3})$ کدام است؟

$$u = 3x - 1 \rightarrow x = \frac{u+1}{3} \rightarrow dx = \frac{du}{3}$$

$$\int (2u + 2 - 1)\sqrt{u} \cdot \frac{du}{3} = \frac{1}{3} \int (2u + 1)u^{\frac{1}{2}} du$$

$$\frac{1}{3} \int (2u^{\frac{3}{2}} + u^{\frac{1}{2}}) du = \frac{1}{3} \left(\frac{2}{\frac{5}{2}} u^{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right)$$

$$\frac{1}{3} \left(\frac{4}{5} \sqrt{(3x-1)^5} + \frac{2}{3} \sqrt{(3x-1)^3} \right)$$

$$F\left(\frac{2}{3}\right) = \frac{1}{3} \left(\frac{4}{5} \times 1 + \frac{2}{3} \right) = \frac{14}{15} \quad F\left(\frac{1}{3}\right) = 0$$

جواب: $\frac{14}{15}$

۹- حاصل انتگرال $\int \frac{x^3 + 1}{x^2} dx$ کدام است؟

$$\int \frac{x^3}{x^2} dx + \int \frac{1}{x^2} dx = \int \underline{x} dx + \int \underline{x^{-2}} dx$$

$$= \frac{x^2}{2} + \frac{x^{-1}}{-1} = \frac{x^2}{2} - \frac{1}{x}$$

۱۰- حاصل انتگرال $\int \frac{1 + \sqrt{x}}{x} dx$ کدام است؟

$$\int \left(\frac{1}{x} + \frac{\sqrt{x}}{x} \right) dx = \int \left(\frac{1}{x} + x^{-\frac{1}{2}} \right) dx$$

$$= \ln|x| + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \ln x + 2\sqrt{x}$$

۱۱- حاصل انتگرال $\int \frac{(x\sqrt{x} + 2\sqrt{x})^2}{x} dx$ کدام است؟

$$\int \frac{(\sqrt{x}(x+2))^2}{x} dx = \int \frac{\cancel{x} (x+2)^2}{\cancel{x}} dx$$

$$\int \underline{\underline{(x+2)^\mu}} dx = \frac{(x+2)^\mu}{\mu}$$

۱۲- اگر $\int \frac{5x^2 + 3x}{\sqrt{x}} dx = \underline{\underline{\sqrt{x} f(x) + c}}$ آنگاه $f(x)$ کدام است؟

$$\int \left(\frac{5x^2}{\sqrt{x}} + \frac{3x}{\sqrt{x}} \right) dx = \int (5x^{\frac{3}{2}} + 3x^{\frac{1}{2}}) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = 2x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$$

$$\sqrt{x} \left(\underline{2x^2 + 2x} \right)$$

$$f = 2x^2 + 2x$$

۱۳- حاصل انتگرال $\int \sqrt{(x^r - \frac{1}{x^r})^2 + 4x^r} dx$ کدام است؟

$$\int \sqrt{x^r + \frac{1}{x^r} - \underbrace{2x^r + 2x^r}_{+4x^r}} dx = \int \sqrt{\left(x^r + \frac{1}{x^r}\right)^2} dx$$

$$= \int (x^r + x^{-r}) dx = \frac{x^{\omega}}{\omega} + \frac{x^{-1}}{-1} = \frac{x^{\omega}}{\omega} - \frac{1}{x}$$

۱۴- حاصل $\int \frac{(x^2 \sqrt{x} + 2\sqrt{x})^4}{x} dx$ برابر کدام است؟

$$\int \frac{(\sqrt{x}(x^2+2))^4}{x} dx = \int \frac{x^2(x^2+2)^4}{x} dx$$

$$\int \underline{x} (x^2+2)^4 \underline{dx} \rightarrow \begin{cases} U = x^2+2 \\ dU = \underline{2x dx} \end{cases} \rightarrow \frac{1}{2} \int U^4 dU$$

$$= \frac{1}{2} \cdot \frac{U^5}{5} = \frac{1}{10} (x^2+2)^5$$

۱۵- اگر $\int \frac{e^{2x} - 1}{e^x - 1} dx = f(x) + x + c$ کدام است $f(x)$ ؟

$$\int \frac{\cancel{e^x - 1} (e^x + 1)}{\cancel{e^x - 1}} dx = \int \underline{\underline{(e^x + 1)}} dx$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int e^{\mu x} dx = \frac{1}{\mu} e^{\mu x}$$

$$\int \frac{1}{e^x} = e^{-x} + x + C$$

$$f(x) = e^x$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

۱۶- حاصل $\int \frac{e^{3x} + 1}{e^{2x} + e^x} dx$ کدام است؟

$$\int \frac{(e^x + 1)(e^{2x} - e^x + 1)}{e^x(e^x + 1)} dx = \int e^{-x}(e^{2x} - e^x + 1) dx$$

$$= \int (e^x - 1 + e^{-x}) dx = e^x - x - e^{-x}$$

۱۷- اگر $\int \frac{x^2 - 2x + 3}{\sqrt{x-1}} dx = \sqrt{x-1} f(x) + c$ آنگاه $f(1)$ کدام است؟

$$\int \frac{x^2 - 2x + 1 + 2}{\sqrt{x-1}} dx = \int \left(\frac{(x-1)^2}{\sqrt{x-1}} + \frac{2}{\sqrt{x-1}} \right) dx$$

$$\int \left((x-1)^{\frac{3}{2}} + 2(x-1)^{-\frac{1}{2}} \right) dx = \frac{(x-1)^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{(x-1)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$\sqrt{x-1} \left(\frac{2}{5} (x-1)^2 + 4 \right) \rightarrow f(1) = 4$$

۱۸- حاصل $\int \frac{1-x}{1-\sqrt{x}} dx$ کدام است؟

$$\int \frac{(1-\sqrt{x})(1+\sqrt{x})}{1-\sqrt{x}} dx = \int (1+x^{\frac{1}{2}}) dx$$

$$= x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = x + \frac{2}{3} x\sqrt{x}$$

۱۹- اگر $\int \frac{x^3 - x}{x + \sqrt{x}} dx = \underline{\underline{x\sqrt{x}f(x)}} + C$ باشد، حاصل $f(0)$ کدام است؟ (C عدد ثابت است).

$$\int \frac{x(x-1)(x+1)}{\sqrt{x}(\sqrt{x}+1)} dx = \int \frac{\cancel{\sqrt{x}} \cdot \sqrt{x} (\sqrt{x}-1)(\cancel{\sqrt{x}+1})(x+1)}{\cancel{\sqrt{x}}(\cancel{\sqrt{x}+1})} dx$$

$$= \int (x - \sqrt{x})(x+1) dx = \int (x^2 + x - x^{\frac{3}{2}} - x^{\frac{1}{2}}) dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - \frac{2}{5} x^{\frac{5}{2}} - \frac{2}{3} x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}} \left(\frac{1}{3} x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{1}{2}} - \frac{2}{5} x - \frac{2}{3} \right) \rightarrow \boxed{f(0) = -\frac{2}{3}}$$

۲-اگر $\int \frac{x(x+2)}{(x+1)^2} dx = \frac{f(x)}{x+1} + c$ باشد، آن گاه ضابطه‌ی $f(x)$ کدام می‌تواند باشد؟

$$\int \frac{x^2 + 2x}{(x+1)^2} dx = \int \frac{x^2 + 2x + 1 - 1}{(x+1)^2} dx = \int \frac{(x+1)^2 - 1}{(x+1)^2} dx$$

$$\int (1 - (x+1)^{-2}) dx = x + \frac{1}{x+1} = \frac{x^2 + x + 1}{x+1}$$

$f = x^2 + x + 1$

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