آموزش ریاضی

انتگرال

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$$\int a\,dx = ax \qquad \int ۵\,dx = ۵x$$

$$\int x^{a}\,dx = \frac{x^{a+1}}{a+1} \qquad \int x^{۵}\,dx = \frac{x^{۶}}{۶}$$

$$\int \frac{dx}{x^{۳}} = \int x^{-۳}\,dx = \frac{x^{-۲}}{-۲} = \frac{-۱}{۲x^{۲}}$$

$$\int \frac{dx}{\sqrt{x}} = \int \frac{dx}{x^{۱/۲}} = \int x^{-۱/۲}\,dx = \frac{x^{۱/۲}}{۱/۲} = ۲\sqrt{x}$$

$$\int (x^3 + 5x + 4)\,dx = \frac{x^4}{4} + 5\,\frac{x^2}{2} + 4x$$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int (x+1)\,x^{-1/2}\,dx = \int \left(x^{1/2} + x^{-1/2}\right)dx = \frac{2}{3}\,x^{3/2} + 2x^{1/2}$$

$$\int \sqrt[5]{x^2}\,dx = \int x^{2/5}\,dx = \frac{5}{7}\,x^{7/5} = \frac{5}{7}\,\sqrt[5]{x^7}$$

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$$\int u^n\,du = \frac{u^{n+1}}{n+1}$$

$$\int 2x(1+x^2)^5\,dx \longrightarrow \left|\begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array}\right. \longrightarrow \int u^5\,du = \frac{u^6}{6} = \frac{(1+x^2)^6}{6}$$

$$\int x^3(1+5x^4)^{10}\,dx \longrightarrow \left|\begin{array}{l} u = 1+5x^4 \\ du = 20x^3\,dx \end{array}\right. \longrightarrow \frac{1}{20}\int u^{10}\,du = \frac{1}{20}\cdot\frac{u^{11}}{11} = \frac{(1+5x^4)^{11}}{220}$$

$$\int \frac{dx}{(\arctan x)^3(1+x^2)} \longrightarrow \left|\begin{array}{l} u = \arctan x \\ du = \frac{1}{1+x^2}\,dx \end{array}\right. \longrightarrow \int \frac{du}{u^3} = \frac{u^{-2}}{-2} = \frac{-1}{2(\arctan x)^2}$$

$$\int \frac{(\ln x+3)^4}{x}\,dx \longrightarrow \left|\begin{array}{l} u=\ln x+3 \\ du=\frac{1}{x}\,dx \end{array}\right. \longrightarrow \int u^4\,du=\frac{u^5}{5}=\frac{(\ln x+3)^5}{5}$$

$$\int e^{2x}(1-3e^{2x})^5\,dx \longrightarrow \left|\begin{array}{l} u=1-3e^{2x} \\ du=-6e^{2x}\,dx \end{array}\right. \longrightarrow \frac{-1}{6}\int u^5\,du=\frac{-1}{6}\cdot\frac{u^6}{6}=\frac{(1-3e^{2x})^6}{-36}$$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx \longrightarrow \left|\begin{array}{l} u=\arcsin x \\ du=\frac{1}{\sqrt{1-x^2}}\,dx \end{array}\right. \longrightarrow \int u\,du=\frac{u^2}{2}=\frac{(\arcsin x)^2}{2}$$

$$\int x\sqrt{x+1}\,dx \longrightarrow \left|\begin{array}{l} u = x+1 \\ x = u-1 \\ du = dx \end{array}\right.$$

$$\int (u-1)\sqrt{u}\,du = \int \left(u^{3/2} - u^{1/2}\right) du = \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2}$$

$$\text{جواب} = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

$$\int x(x-1)^9\,dx \rightarrow \left|\begin{array}{l} u = x-1 \\ x = u+1 \\ dx = du \end{array}\right.$$

$$\int (u+1)\,u^9\,du = \int \left(u^{10} + u^9\right) du = \frac{u^{11}}{11} + \frac{u^{10}}{10}$$

جواب $= \dfrac{(x-1)^{11}}{11} + \dfrac{(x-1)^{10}}{10}$

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$$\int \frac{du}{u} = Ln|u|$$

$$\int \frac{dx}{x+1} \Rightarrow \left| \begin{array}{l} u = x+1 \\ du = dx \end{array} \right. \Rightarrow \int \frac{du}{u} = Ln|u| = Ln|x+1|$$

$$\int \frac{x^{۲}dx}{x^{۳}+1} \Rightarrow \left| \begin{array}{l} u = x^{۳}+1 \\ du = ۳x^{۲}dx \end{array} \right. \Rightarrow \frac{۱}{۳}\int \frac{du}{u} = \frac{۱}{۳} Ln|x^{۳}+1|$$

$$\int \frac{dx}{\arctan x\,(1+x^2)} \longrightarrow \left| \begin{array}{l} u = \arctan x \\ du = \dfrac{1}{1+x^2}\,dx \end{array} \right.$$

$$\int \frac{du}{u} = \ln|u| = \ln|\arctan x|$$

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$$\int \frac{dx}{x\ln x} \longrightarrow \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array} \right.$$

$$\longrightarrow \int \frac{du}{u} = \ln|u| = \ln|\ln x|$$

$$\int e^{u}\,du = e^{u} \qquad \int a^{u}\,du = \frac{a^{u}}{\ln a}$$

$$\int \cos x \cdot e^{\sin x}\,dx \rightarrow \left| \begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array} \right. \rightarrow \int e^{u}\,du = e^{u} = e^{\sin x}$$

$$\int \frac{2^{\arcsin x}}{\sqrt{1-x^{2}}}\,dx \rightarrow \left| \begin{array}{l} u = \arcsin x \\ du = \frac{1}{\sqrt{1-x^{2}}}\,dx \end{array} \right. \rightarrow \int 2^{u}\,du = \frac{2^{u}}{\ln 2} = \frac{2^{\arcsin x}}{\ln 2}$$

$$\int \sin u \, du = -\cos u \qquad \int \cos u \, du = \sin u$$

$$\int x \sin(x^2+1)\,dx \to \left| \begin{array}{l} u = x^2+1 \\ du = 2x\,dx \end{array} \right. \to \frac{1}{2}\int \sin u \, du = \frac{-1}{2}\cos(x^2+1)$$

$$\int \frac{\cos(\ln x + 4)}{x}\,dx \to \left| \begin{array}{l} u = \ln x + 4 \\ du = \frac{1}{x}\,dx \end{array} \right. \to \int \cos u \, du = \sin(\ln x + 4)$$

$$\int \frac{\sin(\arctan x)}{1+x^2}\,dx \to \left| \begin{array}{l} u = \arctan x \\ du = \frac{1}{1+x^2}\,dx \end{array} \right. \to \int \sin u \, du = -\cos(\arctan x)$$

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$$\int \cos^3 2x \cdot \sin 2x \, dx \Rightarrow \begin{cases} u = \cos 2x \\ du = -2 \sin 2x \, dx \end{cases}$$

$$\rightarrow \frac{-1}{2} \int u^3 \, du = \frac{-1}{2} \cdot \frac{u^4}{4} = \frac{-1}{8} (\cos 2x)^4$$

جواب $= \frac{-1}{8} \cos^4 2x$

$$\int \sin^2 5x\,dx = \int \left(\frac{1-\cos 10x}{2}\right)dx = \frac{1}{2}\left(x - \frac{1}{10}\sin 10x\right)$$

$$\int \cos^2 7x\,dx = \int \left(\frac{1+\cos 14x}{2}\right)dx = \frac{1}{2}\left(x + \frac{1}{14}\sin 14x\right)$$

$$\int \sin^3 x\,dx = \int \sin x \cdot \sin^2 x\,dx = \int \sin x\,(1-\cos^2 x)\,dx$$

$$= \int \sin x\,dx - \int \sin x \cos^2 x\,dx$$

$$\left|\begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array}\right.$$

$$= -\cos x + \frac{\cos^3 x}{3}$$

$$\int \cos^5 x\,dx = \int \cos x\,(1-\sin^2 x)^2\,dx$$

$$\int \cos x\,(1+\sin^4 x - 2\sin^2 x)\,dx \longrightarrow \begin{cases} u = \sin x \\ du = \cos x\,dx \end{cases}$$

$$= \int (\cos x + \sin^4 x \cos x - 2\sin^2 x \cos x)\,dx$$

$$= \sin x + \frac{\sin^5 x}{5} - 2\,\frac{\sin^3 x}{3}$$

۱) $\sin a \cdot \cos b = \frac{1}{۲}\left(\sin(a+b) + \sin(a-b)\right)$

۲) $\cos a \cdot \cos b = \frac{1}{۲}\left(\cos(a-b) + \cos(a+b)\right)$

۳) $\sin a \cdot \sin b = \frac{1}{۲}\left(\cos(a-b) - \cos(a+b)\right)$

$$\int \sin ۵x \cdot \cos ۲x \, dx = \frac{1}{۲}\int \left(\sin ۷x + \sin ۳x\right) dx$$

$$= \frac{1}{۲}\left(\frac{-1}{۷}\cos ۷x - \frac{1}{۳}\cos ۳x\right) = \frac{-1}{۱۴}\cos ۷x - \frac{1}{۶}\cos ۳x$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x|$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x|$$

$$\int x^2 \tan(x^3+1) \, dx \rightarrow \left| \begin{array}{l} u = x^3+1 \\ du = 3x^2 dx \end{array} \right. \rightarrow \frac{1}{3} \int \tan u \, du = \frac{1}{3} \ln|\cos(x^3+1)|$$

$$\int \frac{\cot(\ln x)}{x} dx \rightarrow \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right. \rightarrow \int \cot u \, du = \ln|\sin(\ln x)|$$

$$\int \tan^2 x\,dx = \int (\tan^2 x + 1 - 1)\,dx = \tan x - x$$

$$\int \tan^4 x\,dx = \int (\tan^4 x + \tan^2 x - \tan^2 x - 1 + 1)\,dx$$

$$= \int \left(\tan^2 x(\tan^2 x + 1) - (1 + \tan^2 x) + 1\right)dx$$

$$= \frac{1}{3}\tan^3 x - \tan x + x$$

$$\int \cot^3 x\,dx = \int (\cot^3 x + \cot x - \cot x)\,dx$$

$$= \int \left(\cot x(\cot^2 x + 1) - \frac{\cos x}{\sin x}\right)dx$$

$$= \frac{-\cot^2 x}{2} - \ln|\sin x|$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{Arctan}\frac{x}{a}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\operatorname{Ln}\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{x^2+9} = \frac{1}{3}\operatorname{Arctan}\frac{x}{3}$$

$$\int \frac{dx}{x^2-9} = \frac{1}{6}\operatorname{Ln}\left|\frac{x-3}{x+3}\right|$$

$$\int \frac{dx}{x^2+5} = \frac{1}{\sqrt{5}}\operatorname{Arctan}\frac{x}{\sqrt{5}}$$

$$\int \frac{dx}{x^2-5} = \frac{1}{2\sqrt{5}}\operatorname{Ln}\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|$$

$$\int \frac{x\,dx}{1+x^2} \longrightarrow \begin{cases} u = x^2 \\ du = 2x\,dx \end{cases}$$

$$\longrightarrow \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\,\mathrm{Arctan}\,\frac{x^2}{1}$$

$$\int \frac{\cos x\,dx}{\sin^2 x + 9} \longrightarrow \begin{cases} u = \sin x \\ du = \cos x\,dx \end{cases}$$

$$\longrightarrow \int \frac{du}{u^2+9} = \frac{1}{3}\,\mathrm{Arctan}\,\frac{x}{3}$$

جواب : $= \frac{1}{3}\,\mathrm{Arctan}\left(\frac{\sin x}{3}\right)$

$$\int \frac{dx}{x(\ln^2 x + 25)} \Longrightarrow \begin{cases} u = \ln x \\ du = \frac{1}{x}\,dx \end{cases}$$

$$\Longrightarrow \int \frac{du}{u^2 + 25} = \frac{1}{5}\operatorname{Arctan}\frac{u}{5}$$

ج. $= \dfrac{1}{5}\operatorname{Arctan}\left(\dfrac{\ln x}{5}\right)$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{ArcSin}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{25-x^2}} = \text{ArcSin}\left(\frac{x}{5}\right)$$

$$\int \frac{dx}{\sqrt{9-x^2}} = \text{ArcSin}\left(\frac{x}{3}\right)$$

$$\int \frac{\cos x\,dx}{\sqrt{100-\sin^2 x}} \rightarrow \begin{cases} u=\sin x \\ du=\cos x\,dx \end{cases}$$

$$\rightarrow \int \frac{du}{\sqrt{100-u^2}} = \arcsin\left(\frac{\sin x}{10}\right)$$

$$\int \frac{du}{(u-a)(u-b)} = \frac{1}{a-b} \ln \left| \frac{u-a}{u-b} \right|$$

$$\int \frac{dx}{(x-5)(x-1)} = \frac{1}{5-1} \ln \left| \frac{x-5}{x-1} \right| = \frac{1}{4} \ln \left| \frac{x-5}{x-1} \right|$$

$$\int \frac{dx}{x^2-5x+6} = \int \frac{dx}{(x-3)(x-2)} = \frac{1}{3-2} \ln \left| \frac{x-3}{x-2} \right|$$

$$\int \frac{\cos x\,dx}{\sin^2 x + 4\sin x - 5} \quad \left|\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}\right.$$

$$\int \frac{du}{u^2+4u-5} = \int \frac{du}{(u+5)(u-1)} = \frac{1}{-5-1}\ln\left|\frac{u+5}{u-1}\right|$$

$$= \frac{-1}{6}\ln\left|\frac{\sin x+5}{\sin x-1}\right| = \frac{1}{6}\ln\left|\frac{\sin x-1}{\sin x+5}\right|$$

$$\int \frac{2x\,dx}{x^2-5x+4} = \int \frac{-2/3}{x-1}\,dx + \int \frac{8/3}{x-4}\,dx$$

$$= -\frac{2}{3}\ln|x-1| + \frac{8}{3}\ln|x-4|$$

$$\boxed{\frac{2x}{(x-1)(x-4)}} = \frac{-2/3}{\underbrace{x-1}_{x=1}} + \frac{8/3}{\underbrace{x-4}_{x=4}}$$

$$\int \frac{x^3}{x+1}\,dx$$

$$\begin{array}{r|l}
x^3 & x+1 \\ \hline
-(x^3+x^2) & x^2-x+1 \\
-x^2 & \\
-(-x^2-x) & \\ \hline
x & \\
-(x+1) & \\ \hline
-1 &
\end{array}$$

$$\int \left(x^2-x+1+\frac{-1}{x+1}\right)dx$$

$$=\frac{x^3}{3}-\frac{x^2}{2}+x-\ln|x+1|$$

$$\int \frac{dx}{x^3+x} = \int \frac{dx}{x(x^2+1)} = \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx$$

$$= \ln x - \frac{1}{2}\ln|x^2+1|$$

$B=0$

$A=-1$

$$\boxed{\frac{1}{x(x^2+1)}} = \frac{1}{\underset{x=0}{x}} + \frac{\overset{-1x+0}{Ax+B}}{x^2+1} = \frac{x^2+1+Ax^2+Bx}{x(x^2+1)} = \frac{x^2(1+A)+Bx+1}{x(x^2+1)}$$

$$\int x \sin x \, dx = -x\cos x + \sin x$$

x		$\sin x$
	$+$	
1		$-\cos x$
	$-$	
0		$-\sin x$

$$\int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x$$

$$\begin{array}{ccc}
x^2 & & \cos x \\
 & + & \\
2x & & \sin x \\
 & - & \\
2 & & -\cos x \\
 & + & \\
0 & & -\sin x
\end{array}$$

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$$\int x^3 e^{2x}\,dx = e^{2x}\left(\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right)$$

$$\begin{array}{ccc}
x^3 & & e^{2x} \\
 & + & \\
3x^2 & & \frac{1}{2}e^{2x} \\
 & - & \\
6x & & \frac{1}{4}e^{2x} \\
 & + & \\
6 & & \frac{1}{8}e^{2x} \\
 & - & \\
0 & & \frac{1}{16}e^{2x}
\end{array}$$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$$

$\ln x$ $\qquad\qquad$ x

$$\boxed{\frac{1}{x} \qquad \frac{x^2}{2}}$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4}$$

$$\int x \,\mathrm{Arctan}\, x \, dx = \frac{x^2}{2}\mathrm{Arctan}\, x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\mathrm{Arctan}\, x - \frac{1}{2}\int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right) dx$$

$\mathrm{Arctan}\, x \qquad x$

$$\boxed{\frac{1}{1+x^2} \qquad \frac{x^2}{2}}$$

$$= \frac{x^2}{2}\mathrm{Arctan}\, x - \frac{1}{2}\left(x - \mathrm{Arctan}\, x\right)$$

$$\int x^{n} \ln x \, dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^{۲}}$$

$$\int x^{۱} \ln x \, dx = \frac{x^{۲}}{۲} \ln x - \frac{x^{۲}}{۴}$$

$$\int x^{۳} \ln x \, dx = \frac{x^{۴}}{۴} \ln x - \frac{x^{۴}}{۱۶}$$

$$\int \operatorname{ArcSin} x \, dx = x \operatorname{ArcSin} x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$\operatorname{ArcSin} x \qquad 1$

$\dfrac{1}{\sqrt{1-x^2}} \qquad x$

$$= x \operatorname{ArcSin} x + \sqrt{1-x^2}$$

$$\begin{cases} u = 1-x^2 \\ du = -2x\,dx \end{cases} \longrightarrow -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = -\sqrt{u} = -\sqrt{1-x^2}$$

$$\int \frac{dx}{e^{x}+1} \times \frac{e^{-x}}{e^{-x}} = \int \frac{e^{-x}\,dx}{1+e^{-x}}$$

$$\begin{cases} u = 1 + e^{-x} \\ du = -e^{-x}\,dx \end{cases} \longrightarrow -\int \frac{du}{u} = -\ln|u|$$

جواب $= -\ln\left|1+e^{-x}\right|$

$$\int \frac{dx}{e^{x}+e^{-x}} = \int \frac{dx}{e^{x}+\frac{1}{e^{x}}} = \int \frac{e^{x}\,dx}{e^{2x}+1}$$

$$\left| \begin{array}{l} u = e^{x} \\ du = e^{x}\,dx \end{array} \right. \longrightarrow \int \frac{du}{u^{2}+1} = \operatorname{Arctan} u$$

پس $= \operatorname{Arctan}(e^{x})$

$$\int x^2\sqrt{x+1}\,dx \longrightarrow \left| \begin{array}{l} u = x+1 \\ x = u-1 \\ dx = du \end{array} \right.$$

$$\int (u-1)^2\sqrt{u}\,du = \int (u^2-2u+1)\,u^{1/2}\,du$$

$$= \int \left(u^{5/2}-2u^{3/2}+u^{1/2}\right)du = \frac{2}{7}u^{7/2}-\frac{4}{5}u^{5/2}+\frac{2}{3}u^{3/2}$$

$$\text{جواب نهایی:}\quad = \frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

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$$\int \frac{dx}{x\ln x - x} = \int \frac{dx}{x(\ln x - 1)}$$

$$\rightarrow \begin{cases} u = \ln x - 1 \\ du = \frac{1}{x}dx \end{cases} \rightarrow \int \frac{du}{u} = \ln|u|$$

جواب $= \ln|\ln x - 1|$

$$\int \frac{dx}{\sqrt{x}\,(x-1)} \longrightarrow \left|\begin{array}{l} u=\sqrt{x} \\ du=\dfrac{1}{2\sqrt{x}}\,dx \end{array}\right.$$

$$2\int \frac{du}{u^2-1} = 2\left(\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|\right)$$

$$= \ln\left|\frac{\sqrt{x}-1}{\sqrt{x}+1}\right|$$