

آموزش ریاضی

انتگرال

Algebra.com

$$\int a \, dx = ax$$

$$\int \omega \, dx = \omega x$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1}$$

$$\int x^a \, dx = \frac{x^y}{y}$$

$$\int \frac{dx}{x^r} = \int x^{-r} \, dx = \frac{x^{-r}}{-r} = \frac{-1}{rx^r}$$

$$\int \frac{dx}{\sqrt{x}} = \int \frac{dx}{x^{1/2}} = \int x^{-1/2} \, dx = \frac{x^{1/2}}{1/2} = 2\sqrt{x}$$

$$\int (x^r + \omega x + r) dx = \frac{x^{r+1}}{r+1} + \omega \frac{x^2}{2} + rx$$

$$\int \frac{x+1}{\sqrt{x}} dx = \int (x+1) x^{-1/2} dx = \int (x^{1/2} + x^{-1/2}) dx = \frac{2}{3} x^{3/2} + 2x^{1/2}$$

$$\int \omega \sqrt{x^r} dx = \int x^{r/2} dx = \frac{\omega}{\frac{r}{2}+1} x^{\frac{r}{2}+1} = \frac{\omega}{\frac{r+2}{2}} \sqrt{x^{r+2}}$$

$$\int U^n du = \frac{U^{n+1}}{n+1}$$

$$\int r x (1+x^r)^a dx \rightarrow \begin{cases} U = 1+x^r \\ du = r x dx \end{cases} \rightarrow \int U^a du = \frac{U^a}{a} = \frac{(1+x^r)^a}{a}$$

$$\int x^p (1+ax^r)^q dx \rightarrow \begin{cases} U = 1+ax^r \\ du = r a x^{r-1} dx \end{cases} \rightarrow \frac{1}{r a} \int U^q du = \frac{1}{r a} \cdot \frac{U^{q+1}}{q+1} = \frac{(1+ax^r)^{q+1}}{r a (q+1)}$$

$$\int \frac{dx}{(\arctan x)^r (1+x^2)} \rightarrow \begin{cases} U = \arctan x \\ du = \frac{1}{1+x^2} dx \end{cases} \rightarrow \int \frac{du}{U^r} = \frac{U^{-r}}{-r} = \frac{-1}{r (\arctan x)^r}$$

$$\int \frac{(\ln x + \mu)^{\omega}}{x} dx \rightarrow \begin{cases} U = \ln x + \mu \\ du = \frac{1}{x} dx \end{cases} \rightarrow \int U^{\omega} du = \frac{U^{\omega+1}}{\omega+1} = \frac{(\ln x + \mu)^{\omega+1}}{\omega+1}$$

$$\int e^{\mu x} (1 - \mu e^{\mu x})^{\omega} dx \rightarrow \begin{cases} U = 1 - \mu e^{\mu x} \\ du = -\mu e^{\mu x} dx \end{cases} \rightarrow \int \frac{-1}{\mu} U^{\omega} du = \frac{-1}{\mu} \cdot \frac{U^{\omega+1}}{\omega+1} = \frac{(1 - \mu e^{\mu x})^{\omega+1}}{-\mu(\omega+1)}$$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx \rightarrow \begin{cases} U = \arcsin x \\ du = \frac{1}{\sqrt{1-x^2}} dx \end{cases} \rightarrow \int U du = \frac{U^2}{2} = \frac{(\arcsin x)^2}{2}$$

$$\int x\sqrt{x+1} dx \rightarrow \begin{cases} u = x+1 \\ x = u-1 \\ du = dx \end{cases}$$

$$\int (u-1)\sqrt{u} du = \int (u^{\frac{\mu}{r}} - u^{\frac{1}{r}}) du = \frac{r}{\omega} u^{\frac{\omega}{r}} - \frac{r}{\mu} u^{\frac{\mu}{r}}$$

$$\therefore \int = \frac{r}{\omega} (x+1)^{\frac{\omega}{r}} - \frac{r}{\mu} (x+1)^{\frac{\mu}{r}}$$

$$\int x(x-1)^9 dx$$

$$\begin{cases} u = x - 1 \\ x = u + 1 \\ dx = du \end{cases}$$

$$\int (u+1)u^9 du = \int (u^{10} + u^9) du = \frac{u^{11}}{11} + \frac{u^{10}}{10}$$

$$\therefore \int x(x-1)^9 dx = \frac{(x-1)^{11}}{11} + \frac{(x-1)^{10}}{10}$$

$$\int \frac{du}{u} = \ln|u|$$

$$\int \frac{dx}{x+1} \rightarrow \left. \begin{array}{l} U = x+1 \\ du = dx \end{array} \right\} \rightarrow \int \frac{du}{u} = \ln|u| = \ln|x+1|$$

$$\int \frac{x^r dx}{x^{\mu} + 1} \rightarrow \left. \begin{array}{l} U = x^{\mu} + 1 \\ du = \mu x^{\mu-1} dx \end{array} \right\} \rightarrow \frac{1}{\mu} \int \frac{du}{u} = \frac{1}{\mu} \ln|x^{\mu} + 1|$$

$$\int \frac{dx}{\arctan x (1+x^2)} \rightarrow \begin{cases} u = \arctan x \\ du = \frac{1}{1+x^2} dx \end{cases}$$

$$\int \frac{du}{u} = \ln|u| = \ln|\arctan x|$$

$$\int \frac{dx}{x \ln x} \rightarrow \begin{cases} u = \ln x \\ du = \frac{1}{x} dx \end{cases}$$

$$\rightarrow \int \frac{du}{u} = \ln |u| = \ln |\ln x|$$

$$\int e^u du = e^u$$

$$\int a^u du = \frac{a^u}{\ln a}$$

$$\int \cos x \cdot e^{\sin x} dx \rightarrow \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array}$$

$$\int e^u du = e^u = e^{\sin x}$$

$$\int \frac{y^{\arcsin x}}{\sqrt{1-x^2}} dx \rightarrow \begin{array}{l} u = \arcsin x \\ du = \frac{1}{\sqrt{1-x^2}} dx \end{array}$$

$$\int y^u du = \frac{y^u}{\ln y} = \frac{y^{\arcsin x}}{\ln y}$$

$$\int \sin u \, du = -\cos u$$

$$\int \cos u \, du = \sin u$$

$$\int x \sin(x^r) \, dx \rightarrow \begin{cases} u = x^r \\ du = r x^{r-1} dx \end{cases} \rightarrow \frac{1}{r} \int \sin u \, du = -\frac{1}{r} \cos(x^r) \checkmark$$

$$\int \frac{\cos(\ln x + r)}{x} \, dx \rightarrow \begin{cases} u = \ln x + r \\ du = \frac{1}{x} dx \end{cases} \rightarrow \int \cos u \, du = \sin(\ln x + r) \checkmark$$

$$\int \frac{\sin(\arctan x)}{1+x^2} \, dx \rightarrow \begin{cases} u = \arctan x \\ du = \frac{1}{1+x^2} dx \end{cases} \rightarrow \int \sin u \, du = -\cos(\arctan x) \checkmark$$

$$\int \cos^{\mu} x \cdot \sin^{\nu} x dx \rightarrow \begin{cases} u = \cos x \\ du = -\sin x dx \end{cases}$$

$$\rightarrow \int \frac{-1}{x} u^{\mu} du = \frac{-1}{x} \cdot \frac{u^{\mu+1}}{\mu+1} = \frac{-1}{\mu+1} (\cos x)^{\mu+1}$$

$$\therefore \int \cos^{\mu} x dx = \frac{-1}{\mu+1} \cos^{\mu+1} x$$

$$\int \sin^2 \omega x dx = \int \left(\frac{1 - \cos 2\omega x}{2} \right) dx = \frac{1}{2} \left(x - \frac{1}{2\omega} \sin 2\omega x \right)$$

$$\int \cos^2 \nu x dx = \int \left(\frac{1 + \cos 2\nu x}{2} \right) dx = \frac{1}{2} \left(x + \frac{1}{2\nu} \sin 2\nu x \right)$$

$$\int \sin^{\mu} x dx = \int \sin x \cdot \sin^{\mu-1} x dx = \int \sin x (1 - \cos^2 x) dx$$

$$= \int \sin x dx - \int \sin x \cos^2 x dx$$

$$\begin{array}{l} u = \cos x \\ du = -\sin x dx \end{array}$$

$$= -\cos x + \frac{\cos^{\mu} x}{\mu}$$

$$\int \cos^{\omega} x dx = \int \cos x (1 - \sin^2 x)^{\nu} dx$$

$$\int \cos x (1 + \sin^{\mu} x - \nu \sin^{\nu} x) dx \rightarrow \begin{cases} u = \sin x \\ du = \cos x dx \end{cases}$$

$$= \int (\cos x + \sin^{\mu} x \cos x - \nu \sin^{\nu} x \cos x) dx$$

$$= \sin x + \frac{\sin^{\omega} x}{\omega} - \nu \frac{\sin^{\mu} x}{\mu}$$

$$1) \sin a \cdot \cos b = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$2) \cos a \cdot \cos b = \frac{1}{2} (\cos(a-b) + \cos(a+b))$$

$$3) \sin a \cdot \sin b = \frac{1}{2} (\cos(a-b) - \cos(a+b))$$

$$\int \sin \omega x \cdot \cos \nu x dx = \frac{1}{2} \int (\sin \sqrt{x} + \sin \sqrt{x}) dx$$

$$= \frac{1}{2} \left(\frac{-1}{\sqrt{x}} \cos \sqrt{x} - \frac{1}{\nu} \cos \sqrt{x} \right) = \frac{-1}{1x} \cos \sqrt{x} - \frac{1}{4} \cos \sqrt{x}$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln |\cos x|$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln |\sin x|$$

$$\int x^\mu \tan(x^\mu + 1) \, dx \rightarrow \begin{cases} U = x^\mu + 1 \\ dU = \mu x^{\mu-1} dx \end{cases} \rightarrow \frac{1}{\mu} \int \tan u \, du = \frac{1}{\mu} \ln |\cos(x^\mu + 1)|$$

$$\int \frac{\cot(\ln x)}{x} \, dx \rightarrow \begin{cases} U = \ln x \\ dU = \frac{1}{x} dx \end{cases} \rightarrow \int \cot u \, du = \ln |\sin(\ln x)|$$

$$\int \tan^{\nu} x dx = \int (\tan^{\nu} x + 1 - 1) dx = \tan x - x \quad \checkmark$$

$$\int \tan^{\mu} x dx = \int (\tan^{\mu} x + \tan^{\mu} x - \tan^{\mu} x - 1 + 1) dx$$

$$= \int (\tan^{\mu} x (\tan^{\mu} x + 1) - (1 + \tan^{\mu} x) + 1) dx$$

$$= \frac{1}{\mu} \tan^{\mu} x - \tan x + x \quad \checkmark$$

$$\int \cot^{\mu} x \, dx = \int (\cot^{\mu} x + \cot x - \cot x) \, dx$$

$$= \int \left(\cot x (\cot^{\mu} x + 1) - \frac{\cos x}{\sin x} \right) dx$$

$$= \frac{-\cot^{\mu} x}{\mu} - \ln |\sin x|$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{Arctan} \frac{x}{a}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right|$$

$$\int \frac{dx}{x^2 + 9} = \frac{1}{\mu} \operatorname{Arctan} \frac{x}{\mu}$$

$$\int \frac{dx}{x^2 - 9} = \frac{1}{9} \ln \left| \frac{x - \mu}{x + \mu} \right|$$

$$\int \frac{dx}{x^2 + \omega} = \frac{1}{\sqrt{\omega}} \operatorname{Arctan} \frac{x}{\sqrt{\omega}}$$

$$\int \frac{dx}{x^2 - \omega} = \frac{1}{\sqrt{\omega}} \ln \left| \frac{x - \sqrt{\omega}}{x + \sqrt{\omega}} \right|$$

$$\int \frac{x dx}{1 + \underline{\underline{x^r}}} \rightarrow \begin{cases} u = \underline{\underline{x^r}} \\ du = r x dx \end{cases}$$

$$\rightarrow \frac{1}{r} \int \frac{du}{1 + u^r} = \frac{1}{r} \operatorname{Arctan} \frac{x^r}{1}$$

$$\int \frac{\cos x dx}{\sin^2 x + 9} \quad \left\{ \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \right.$$

$$\rightarrow \int \frac{du}{u^2 + 9} = \frac{1}{\mu} \text{Arc} \left[\tan \frac{x}{\mu} \right]$$

$$\therefore \int \frac{\cos x dx}{\sin^2 x + 9} = \frac{1}{\mu} \text{Arc} \left[\tan \left(\frac{\sin x}{\mu} \right) \right]$$

$$\int \frac{dx}{x(\ln^2 x + 1)} \rightarrow \begin{cases} u = \ln x \\ du = \frac{1}{x} dx \end{cases}$$

$$\rightarrow \int \frac{du}{u^2 + 1} = \frac{1}{1} \operatorname{Arctan} \frac{u}{1}$$

$$\therefore \int \frac{dx}{x(\ln^2 x + 1)} = \frac{1}{1} \operatorname{Arctan} \left(\frac{\ln x}{1} \right)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \text{Arc Sin} \left(\frac{x}{a} \right)$$

$$\int \frac{dx}{\sqrt{10 - x^2}} = \text{Arc Sin} \left(\frac{x}{\sqrt{10}} \right)$$

$$\int \frac{dx}{\sqrt{9 - x^2}} = \text{Arc Sin} \left(\frac{x}{3} \right)$$

$$\int \frac{\cos x dx}{\sqrt{100 - \sin^2 x}} \quad \left| \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \right.$$

$$\rightarrow \int \frac{du}{\sqrt{100 - u^2}} = \text{ARC SIN} \left(\frac{\sin x}{10} \right)$$

$$\int \frac{du}{(u-a)(u-b)} = \frac{1}{a-b} \ln \left| \frac{u-a}{u-b} \right|$$

$$\int \frac{dx}{(x-a)(x-1)} = \frac{1}{a-1} \ln \left| \frac{x-a}{x-1} \right| = \frac{1}{r} \ln \left| \frac{x-a}{x-1} \right|$$

$$\int \frac{dx}{x^2 - \alpha x + \gamma} = \int \frac{dx}{(x-r)(x-r)} = \frac{1}{r-r} \ln \left| \frac{x-r}{x-r} \right|$$

$$\int \frac{\cos x dx}{\sin^2 x + \omega \sin x - 1} \quad \left| \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \right.$$

$$\int \frac{du}{u^2 + \omega u - 1} = \int \frac{du}{(u + \omega)(u - 1)} = \frac{1}{-\omega - 1} \ln \left| \frac{u + \omega}{u - 1} \right|$$

$$= \frac{-1}{\omega} \ln \left| \frac{\sin x + \omega}{\sin x - 1} \right| = \frac{1}{\omega} \ln \left| \frac{\sin x - 1}{\sin x + \omega} \right|$$

$$\int \frac{\mu x dx}{x^2 - \alpha x + \beta} = \int \frac{-\frac{\mu}{\mu}}{x-1} dx + \int \frac{\frac{\mu}{\mu}}{x-\beta} dx$$

$$= \frac{-\mu}{\mu} \ln|x-1| + \frac{\mu}{\mu} \ln|x-\beta|$$

$$\boxed{\frac{\mu x}{(x-1)(x-\beta)}} = \frac{-\frac{\mu}{\mu}}{x-1} + \frac{\frac{\mu}{\mu}}{x-\beta}$$

$\swarrow \Delta x=1$ $\swarrow \Delta x=\beta$

$$\int \frac{x^\mu}{x+1} dx \quad \rightarrow \quad \begin{array}{r} x^\mu \quad | \quad x+1 \\ \hline x^\mu + x^\mu \\ \hline \end{array}$$

$$\int \left(x^\mu - x+1 + \frac{-1}{x+1} \right) dx$$

$$\begin{array}{r} -x^\mu \\ \hline \end{array}$$

$$\begin{array}{r} -x^\mu - x \\ + \quad + \\ \hline \end{array}$$

$$x$$

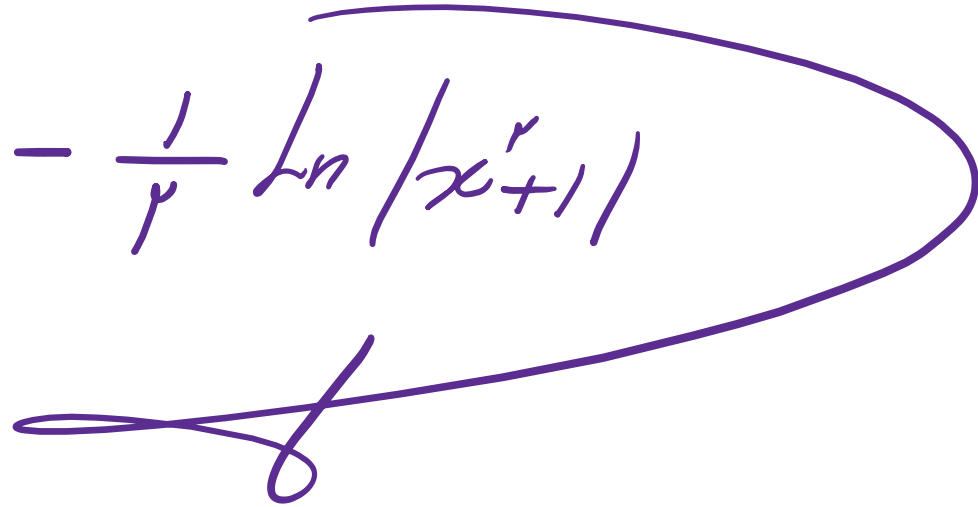
$$\begin{array}{r} -x+1 \\ \hline \end{array}$$

$$-1$$

$$= \frac{x^\mu}{\mu} - \frac{x^\mu}{\mu} + x - \ln|x+1|$$

$$\int \frac{dx}{x^{\mu} + x} = \int \frac{dx}{x(x^{\nu} + 1)} = \int \left(\frac{1}{x} - \frac{x}{x^{\nu} + 1} \right) dx$$

$$= \ln|x| - \frac{1}{\nu} \ln|x^{\nu} + 1|$$

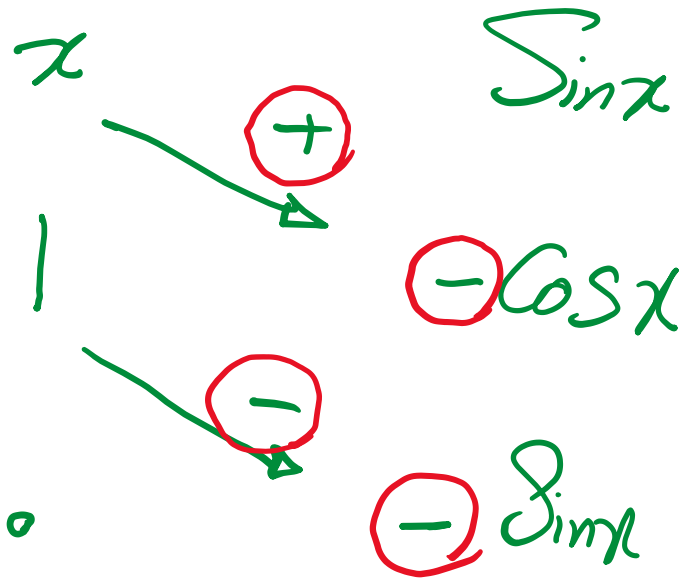


$$\frac{\textcircled{1}}{x(x^{\nu} + 1)} = \frac{1}{x} + \frac{-1x + 0}{x^{\nu} + 1} = \frac{x^{\nu} + 1 + Ax^{\nu} + Bx}{x(x^{\nu} + 1)} = \frac{x^{\nu}(1+A) + Bx + 1}{x(x^{\nu} + 1)}$$

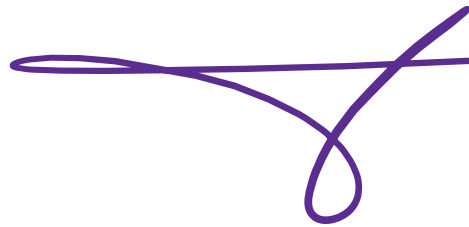
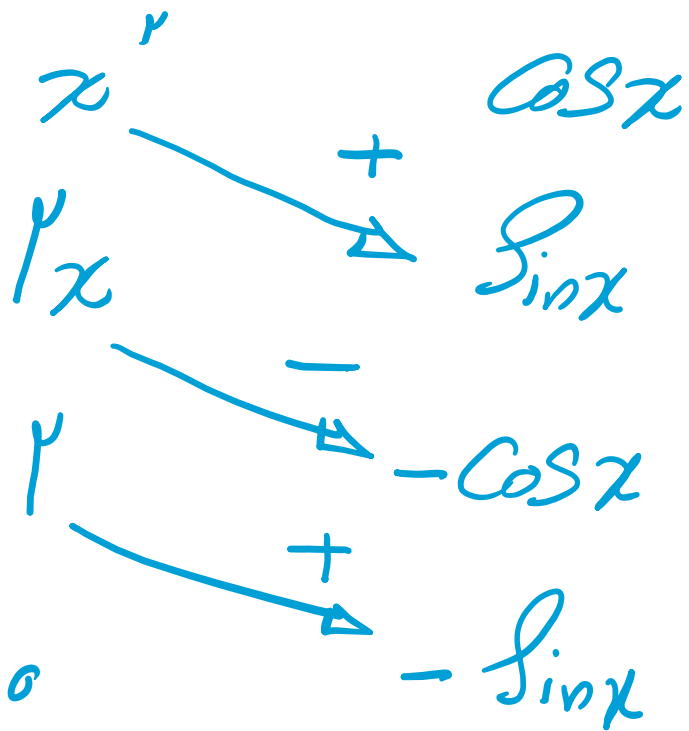
$A = -1$ and $B = 0$ are indicated by arrows pointing to the coefficients in the numerator of the final fraction.

A pink arrow points from the constant term '1' in the numerator of the second fraction to the text $x=0$.

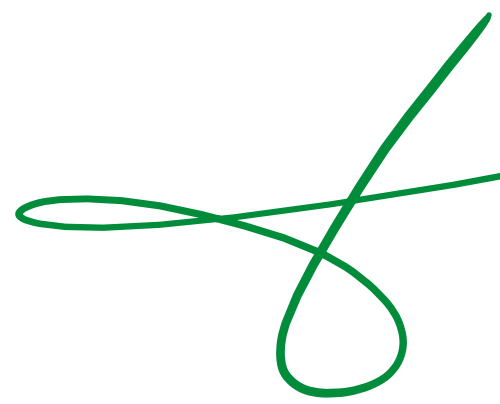
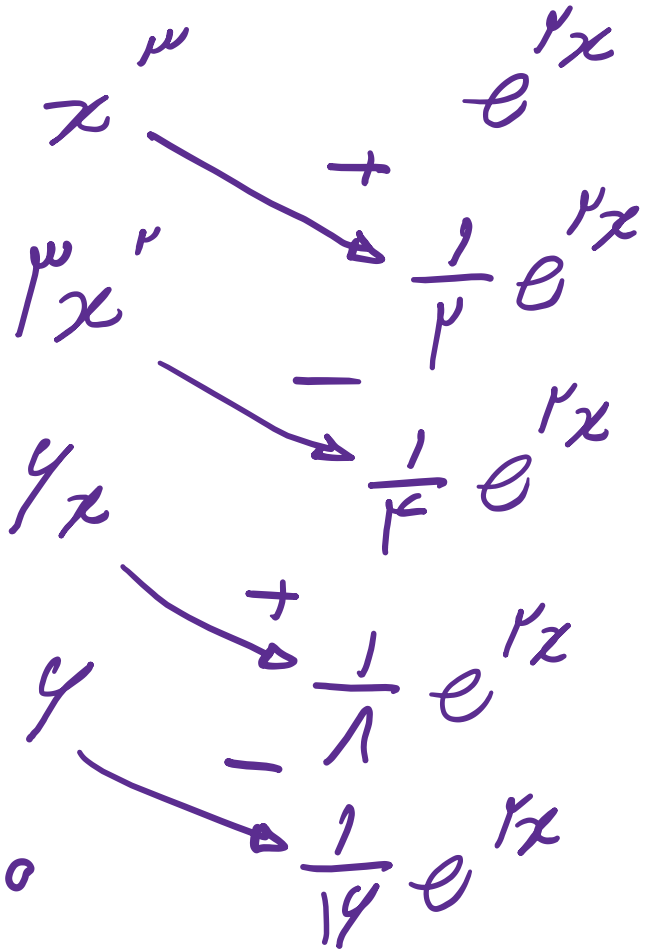
$$\int \underline{x} \underline{\sin x} dx = -x \cos x + \sin x$$



$$\int x^r \cos x dx = x^r \sin x + r x \cos x - r \sin x$$

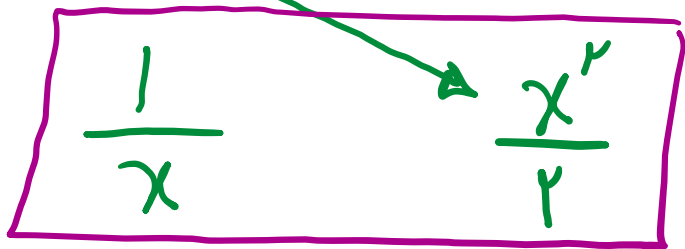


$$\int x^\mu e^{\lambda x} dx = e^{\lambda x} \left(\frac{1}{\lambda} x^\mu - \frac{\mu}{\lambda^2} x^{\mu-1} + \frac{\mu(\mu-1)}{\lambda^3} x^{\mu-2} - \frac{\mu(\mu-1)(\mu-2)}{\lambda^4} x^{\mu-3} + \dots \right)$$



$$\int x \ln x \, dx = \frac{x^y}{y} \ln x - \int \frac{x}{y} \, dx$$

$\ln x$ x



$$= \frac{x^y}{y} \ln x - \frac{x^y}{y}$$

$$\int x \operatorname{Arctan} x \, dx = \frac{x^y}{y} \operatorname{Arctan} x - \int \frac{x^{y+1} - 1}{1+x^y} dx$$

$\operatorname{Arctan} x$

$$= \frac{x^y}{y} \operatorname{Arctan} x - \frac{1}{y} \int \left(\frac{x^{y+1}}{x^{y+1}} - \frac{1}{x^{y+1}} \right) dx$$

$\frac{1}{1+x^y}$	$\frac{x^y}{y}$
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$$= \frac{x^y}{y} \operatorname{Arctan} x - \frac{1}{y} (x - \operatorname{Arctan} x)$$

$$\int x^n \ln x \, dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2}$$

$$\int x^{(1)} \ln x \, dx = \frac{x^{(1)}}{1} \ln x - \frac{x^{(1)}}{1^2}$$

$$\int x^{(f)} \ln x \, dx = \frac{x^{(f)}}{f} \ln x - \frac{x^{(f)}}{f^2}$$

$$\int \text{Arc Sin } x \, dx = x \text{Arc Sin } x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$$\text{Arc Sin } x \quad 1 \quad = x \text{Arc Sin } x + \sqrt{1-x^2}$$

$$\frac{1}{\sqrt{1-x^2}} \quad x$$

$$\begin{aligned} & \left. \begin{array}{l} U=1-x^2 \\ dU=-2x \, dx \end{array} \right\} \rightarrow \int \frac{-1}{\sqrt{U}} \frac{dU}{2} = -\frac{1}{2} \int U^{-1/2} dU = -\sqrt{U} = -\sqrt{1-x^2} \end{aligned}$$

$$\int \frac{dx}{e^x + 1} \times \frac{e^{-x}}{e^{-x}} = \int \frac{e^{-x} dx}{1 + e^{-x}}$$

$$\begin{aligned} U &= 1 + e^{-x} \\ du &= -e^{-x} dx \end{aligned} \rightarrow$$

$$= \int \frac{du}{u} = -\ln |u|$$

$$\therefore \int \frac{dx}{e^x + 1} = -\ln |1 + e^{-x}|$$

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{e^x dx}{e^{2x} + 1}$$

$$\begin{aligned} | u &= e^x \\ | du &= e^x dx \end{aligned}$$

$$\rightarrow \int \frac{du}{u^2 + 1} = \text{Arctan } u$$

$$\therefore \int \frac{dx}{e^x + e^{-x}} = \text{Arctan}(e^x)$$

$$\int x^r \sqrt{x+1} dx \rightarrow \begin{cases} U = x+1 \\ x = U-1 \\ dx = dU \end{cases}$$

$$\int (U-1)^r \sqrt{U} dU = \int (U^r - rU^{r-1}) U^{1/2} dU$$

$$= \int (U^{r+1/2} - rU^{r-1/2}) dU = \frac{r+1}{r+3/2} U^{r+3/2} - \frac{r}{r-1/2} U^{r+1/2} + \frac{r}{r+1/2} U^{r-1/2}$$

$$\therefore \int = \frac{r+1}{r+3/2} (x+1)^{r+3/2} - \frac{r}{r-1/2} (x+1)^{r+1/2} + \frac{r}{r+1/2} (x+1)^{r-1/2}$$

$$\int \frac{dx}{x \ln x - x} = \int \frac{dx}{x (\ln x - 1)}$$

$$\rightarrow \begin{cases} u = \ln x - 1 \\ du = \frac{1}{x} dx \end{cases}$$

$$\rightarrow \int \frac{du}{u} = \ln |u|$$

$$\therefore \int \frac{dx}{x \ln x - x} = \ln |\ln x - 1|$$

$$\int \frac{dx}{\sqrt{x}(x-1)} \rightarrow \begin{cases} U = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}} dx \end{cases}$$

$$\int \frac{du}{u^2-1} = \frac{1}{2} \left(\frac{1}{u} \ln \left| \frac{u-1}{u+1} \right| \right)$$

$$\int \frac{1}{\sqrt{x}(x-1)} = \ln \left| \frac{\sqrt{x-1}}{\sqrt{x+1}} \right|$$

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