

آموزش توابع چند متغیره

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Algebra.com

$$\lim_{x \rightarrow 1} x^\mu + \omega x = 1 + \omega = \gamma$$

$$\lim_{(x,y) \rightarrow (k,r)} x^\nu y^\mu + l^\mu x + l^\nu y = \nu + \mu + 1 = \nu$$

$$\lim_{x,y \rightarrow (0,0)} \frac{\omega^x + \omega^y}{\cos x + \cos y} = \frac{1+1}{1+1} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin \Delta x}{\tan \Delta x} \sim \lim_{x \rightarrow 0} \frac{\Delta x}{\Delta x} = \frac{\Delta x}{\Delta x}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow r}} \frac{\sin(\Delta xy)}{\Delta x} \sim \lim_{\substack{x \rightarrow 0 \\ y \rightarrow r}} \frac{\Delta xy}{\Delta x} = \frac{\Delta xy}{\Delta} = \frac{\Delta y}{\Delta}$$

$$\lim_{\substack{xy \rightarrow 1 \\ x \rightarrow 1 \\ y \rightarrow r}} \frac{\text{Arc Sin}(xy-1)}{\text{Arctan}(\Delta xy-1)} \sim \lim_{\substack{xy \rightarrow 1 \\ x \rightarrow 1 \\ y \rightarrow r}} \frac{xy-1}{\Delta xy-1} = \frac{1}{r}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sqrt{x+y} - \sqrt{y}}{x}$$

$$\lim_{y \rightarrow 0} \frac{\sqrt{x+y} - \sqrt{y}}{x} = \frac{\sqrt{x}}{x} \rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{x}}{x} = \infty$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+y} - \sqrt{y}}{x} = \frac{0}{0} \text{ HoP} \rightarrow \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{x+y}}}{1} = \frac{1}{\sqrt{y}}$$

$$\lim_{y \rightarrow 0} \frac{1}{\sqrt{y}} = \infty$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 1}} \frac{\sqrt{x+y} - \sqrt{y}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+y} - \sqrt{y}}{x} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{x+y}} - 0}{1} = \frac{1}{\sqrt{y}} \xrightarrow{y \rightarrow 1} \lim_{y \rightarrow 1} \frac{1}{\sqrt{y}} = \frac{1}{y}$$

$$\lim_{y \rightarrow 1} \frac{\sqrt{x+y} - \sqrt{y}}{x} = \frac{\sqrt{x+1} - 1}{x} \xrightarrow{x \rightarrow 0} \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{x+1}}}{1} = \frac{1}{y}$$

$$\therefore \text{جواب} = \frac{1}{y}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 1}} \frac{\sqrt{x+y} - \sqrt{y}}{x} \times \frac{\sqrt{x+y} + \sqrt{y}}{\sqrt{x+y} + \sqrt{y}}$$

$$= \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 1}} \frac{\cancel{x+y} - \cancel{y}}{x (\sqrt{x+y} + \sqrt{y})} = \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 1}} \frac{1}{\sqrt{x+y} + \sqrt{y}} = \frac{1}{4}$$

$$\lim_{x, y \rightarrow 1} \frac{x^\mu - y^\mu}{x - y} = \frac{0}{0}$$

$$\rightarrow \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{\cancel{(x-y)}(x^r + xy^r + y^r)}{\cancel{x-y}} = \mu$$

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{x^\mu - y^\mu}{x - y}$$

$$\begin{aligned} x=1 \rightarrow \lim_{y \rightarrow 1} \frac{1 - y^\mu}{1 - y} = \frac{0}{0} &\xrightarrow{\text{HOP}} \lim_{y \rightarrow 1} \frac{-\mu y^{\mu-1}}{-1} = \mu \quad \checkmark \end{aligned}$$

$$\begin{aligned} y=1 \rightarrow \lim_{x \rightarrow 1} \frac{x^\mu - 1}{x - 1} = \frac{0}{0} &\xrightarrow{\text{HOP}} \lim_{x \rightarrow 1} \frac{\mu x^{\mu-1}}{1} = \mu \quad \checkmark \end{aligned}$$

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{x^r - y^r}{\sqrt{x} - \sqrt{y}} = \frac{0}{0}$$

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{x^r - y^r}{\sqrt{x} - \sqrt{y}} \times \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{\cancel{(x-y)}(x+y)(\sqrt{x} + \sqrt{y})}{\cancel{x-y}} = r$$

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{x^r - y^r}{\sqrt{x} - \sqrt{y}}$$

$$\underline{x=1} \rightarrow \lim_{y \rightarrow 1} \frac{1 - y^r}{1 - \sqrt{y}} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{y \rightarrow 1} \frac{-ry}{\frac{1}{2\sqrt{y}}} = r \quad \checkmark$$

$$\underline{y=1} \rightarrow \lim_{x \rightarrow 1} \frac{x^r - 1}{\sqrt{x} - 1} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{x \rightarrow 1} \frac{rx}{\frac{1}{2\sqrt{x}}} = r \quad \checkmark$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{y^r - x^r}{x^r + xy} \quad y = \mu x$$

$$\lim_{x \rightarrow 0} \frac{\mu^r x^r - x^r}{x^r + \mu x^r} = \lim_{x \rightarrow 0} \frac{-\mu x^r}{\mu x^r} = -\frac{\mu}{\mu}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^r y^r}{x^r + y^r} \quad y = mx$$

$$\lim_{x \rightarrow 0} \frac{x^r \cdot m^r x^r}{x^r + m^r x^r} = \lim_{x \rightarrow 0} \frac{x^r \cdot m^r x^r}{x^r (1 + m^r)} = \lim_{x \rightarrow 0} \frac{m^r x^r}{1 + m^r}$$

إذن

$$= 0$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x-y}{x+y} \quad y = \frac{1}{\mu} x$$

$$\lim_{x \rightarrow 0} \frac{x - \frac{1}{\mu} x}{x + \frac{1}{\mu} x} = \lim_{x \rightarrow 0} \frac{\frac{1}{\mu} x}{\frac{\mu}{\mu} x} = \frac{1}{\mu}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy}{\sqrt{x^2 + y^2}} \quad \rightarrow \quad \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$\lim_{r \rightarrow 0} \frac{r \cos \theta \cdot r \sin \theta}{\sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}} = \lim_{r \rightarrow 0} \frac{r^2 \sin \theta \cdot \cos \theta}{r} = \lim_{r \rightarrow 0} r \sin \theta \cos \theta$$

$$= 0$$

$$f(x,y) = \begin{cases} \frac{x^\mu - y^\mu}{x-y} & (x,y) \neq (1,1) \\ a & (x,y) = (1,1) \end{cases}$$

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{x^\mu - y^\mu}{x-y} = \lim_{\substack{x \rightarrow 1 \\ y \rightarrow 1}} \frac{(x-y)(x^{\mu-1} + x^{\mu-2}y + \dots + y^{\mu-1})}{x-y} = \mu$$

سواء
0

$$a = \mu$$

$$f(x,y) = \frac{x+y}{1-x-y} \quad (x,y) \neq (0,0)$$

a

$$(x,y) \neq (0,0)$$

از این سوال به
 صفحه 5

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x+y}{1-x-y} = \text{linear}$$

$$x=0 \rightarrow \lim_{y \rightarrow 0} \frac{y}{-y} = -1 \checkmark$$

$$y=0 \rightarrow \lim_{x \rightarrow 0} \frac{x}{1-x} = \frac{1}{1} \checkmark$$

$$f(x,y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ a & (x,y) = (0,0) \end{cases}$$

$a=0$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 y^2}{x^2 + y^2} \xrightarrow[\substack{x=r \cos \theta \\ y=r \sin \theta}]{r \rightarrow 0} \lim_{r \rightarrow 0} \frac{r^2 \cos^2 \theta \cdot r^2 \sin^2 \theta}{r^2 \cos^2 \theta + r^2 \sin^2 \theta}$$

$$= \lim_{r \rightarrow 0} \frac{r^2 \cdot r^2 \cdot \sin^2 \theta \cdot \cos^2 \theta}{r^2} = \lim_{r \rightarrow 0} r^2 \cos^2 \theta \sin^2 \theta = 0$$

$$f(x,y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ a & (x,y) = (0,0) \end{cases}$$

به ازای کدام مقدار a تابع با ضابطه

در $(0,0)$ پیوسته است؟

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 - y^2}{x^2 + y^2} = \text{حدا$$

$$\begin{aligned} x=0 \rightarrow \lim_{y \rightarrow 0} \frac{-y^2}{y^2} &= -1 \quad \checkmark & y=0 \rightarrow \lim_{x \rightarrow 0} \frac{x^2}{x^2} &= 1 \quad \checkmark \end{aligned}$$

همچون مقدار a پیوسته نخواهد بود.

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