

آموزش توابع چند متغیره

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$$f(x_0 + \Delta x, y_0 + \Delta y) = f(x_0, y_0) + f'_x \Delta x + f'_y \Delta y$$

$$\sqrt{(2.01)^2 - (1.99)^2}$$

$$x_0 = 2$$

$$y_0 = 1$$

$$\Delta x = 0.01$$

$$\Delta y = -0.01$$

$$f = \sqrt{x^2 - y^2} \rightarrow f'_x = \frac{2x}{2\sqrt{x^2 - y^2}}, \quad f'_y = \frac{-2y}{2\sqrt{x^2 - y^2}}$$

$$\text{حوا} = \sqrt{2^2 - 1^2} + \frac{2}{2} \times \frac{1}{100} + \frac{-1}{2} \times \frac{-1}{100}$$

$$= 1 + \frac{1}{100} + \frac{1}{100} = 1 + \frac{2}{100}$$

$$\sqrt{(0.91)^x + 1(1.01)^y}$$

$$x_0 = 1$$

$$y_0 = 1$$

$$\Delta x = -0.01$$

$$\Delta y = 0.01$$

$$f = \sqrt{x^x + 1y^y}$$

$$f'_x = \frac{x x^{x-1}}{\sqrt{x^x + 1y^y}}$$

$$f'_y = \frac{y y^{y-1}}{\sqrt{x^x + 1y^y}}$$

$$\Delta f \approx \sqrt{1+1} + \frac{x}{y} \times \frac{-0.01}{100} + \frac{1}{y} \times \frac{0.01}{100}$$

$$\sqrt{\omega (1,94)^{\mu} + \omega (1,02)^{\nu}}$$

$$x_0 = \mu$$

$$y_0 = 1$$

$$\Delta x = -0,01$$

$$\Delta y = 0,02$$

$$f(x) = \sqrt{\omega x^{\mu} + \omega y^{\nu}}$$

$$f'_x = \frac{\omega x^{\mu-1}}{\omega \sqrt{\omega x^{\mu} + \omega y^{\nu}}}$$

$$f'_y = \frac{\omega y^{\nu-1}}{\omega \sqrt{\omega x^{\mu} + \omega y^{\nu}}}$$

$$\Delta f \approx \sqrt{\omega \mu + \omega} + \frac{\omega \mu}{\omega \times 14} \times \frac{-1}{100} + \frac{\omega \nu}{\omega \times 14} \times \frac{1}{100}$$

$$f = x^2 + y^2 - 4y + 2x + 3$$

$$f'_x = 0 \rightarrow 2x + 2 = 0 \rightarrow x = -1 \quad \checkmark$$

$$f'_y = 0 \rightarrow 2y - 4 = 0 \rightarrow y = 2 \quad \checkmark$$

نقطه بحرانی

$$f = 1 + 4 - 4 - 2 + 3 = -10 + 11 = 1$$

$$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$$\Delta > 0 \begin{cases} f_{xx} > 0 \rightarrow \text{min} \\ f_{xx} < 0 \rightarrow \text{max} \end{cases}$$

$$\Delta < 0 \rightarrow \text{نقطه سرجی}$$

$$f = 1 - 1^u x^v - 1^u y^v + 4x + 4y$$

$$f'_x = 0 \rightarrow -4x + 4 = 0 \rightarrow x = 1$$

$$f'_y = 0 \rightarrow -4y + 4 = 0 \rightarrow y = 1$$

$$f = 1 - 1^u - 1^u + 4 + 4 = V_{\max}$$

$$f_{xx} = -4$$

$$f_{yy} = -4$$

$$f_{xy} = 0$$

$$\Delta = f_{xx} \cdot f_{yy} - (f_{xy})^2 = (-4)(-4) - 0^2 = 16 > 0$$

$$f_{xx} = -4 \rightarrow \Delta > 0, f_{xx} < 0 \rightarrow \max$$

$$f = x^2 - 1x + y^2 - 2y + 1$$

$$f'_x = 2x - 1 = 0 \rightarrow 2x = 1 \rightarrow x = \pm 1$$

$$f'_y = 2y - 2 = 0 \rightarrow 2y = 2 \rightarrow y = 1$$

$$f_{xx} = 2 \quad f_{yy} = 2 \quad f_{xy} = 0$$

$$\Delta = (2)(2) - 0 = 4$$

$$\left. \begin{array}{l} x=1 \\ \Delta=4 \\ f_{xx}=2 \end{array} \right\} \begin{array}{l} \text{min} \\ \underline{f_{xx}} \end{array}$$

$$\left. \begin{array}{l} x=-1 \\ \Delta=4 \\ f_{xx}=2 \end{array} \right\} \begin{array}{l} \text{min} \\ \underline{f_{xx}} \end{array}$$

$$\begin{array}{l} \text{min} \\ C_i \\ \hline A \left| \begin{array}{l} 1 \\ 1 \end{array} \right. \\ B \left| \begin{array}{l} -1 \\ 1 \end{array} \right. \end{array}$$



$$Z = 1x^2 - xy + y^2 + 1x + 1y$$

$$Z'_x = 2x - y = 0 \rightarrow y = 2x = 1 \left( \frac{-1}{1} \right) \rightarrow y = -\frac{1}{1} \checkmark$$

$$Z'_y = -x + 2y + 1 = 0 \rightarrow -x + 1x + 1 = 0 \rightarrow x = -\frac{1}{1} \checkmark$$

$$Z_{xx} = 2 \quad Z_{yy} = 2 \quad Z_{xy} = -1$$

$$\Delta = (2)(2) - (-1) = 4 - (-1) = 5 > 0 \rightarrow Z_{xx} = 2 > 0$$

$\rightarrow \Delta > 0, Z_{xx} > 0 \rightarrow \min f$

نقاط بحرانی تابع  $z = 2x^3 - 6xy + 3y^2$  چگونه اند؟

$$z'_x = 6x^2 - 6y = 0 \rightarrow y = x^2$$
$$z'_y = -6x + 6y = 0 \rightarrow x = y$$

$\left| \begin{array}{c} 0 \\ 1 \end{array} \right|$

$$z_{xx} = 12x \quad z_{yy} = 6 \quad z_{xy} = -6$$

$$\Delta = (12x)(6) - (-6)^2 = 72x - 36$$

$x=1 \rightarrow \Delta = 36 > 0 \rightarrow z_{xx} = 12 > 0 \rightarrow \text{min} \checkmark$

$x=0 \rightarrow \Delta = -36 < 0 \rightarrow \text{Saddle Point}$

نقطه‌ی بحرانی تابع  $f(x, y) = x^3 + 3x^2 + 4xy + y^2$  چگونه است؟

$$f'_x = 3x^2 + 4x + 4y = 0 \quad x = 0 \quad / \quad x = \frac{2}{3}$$

$$f'_y = 4x + 2y = 0 \rightarrow y = -2x \rightarrow 3x^2 + 4x - 1x = 0 \rightarrow 3x^2 - 1x = 0$$

$$f_{xx} = 6x + 4 \quad f_{yy} = 2 \quad f_{xy} = 4$$

$$\Delta = (6x + 4)(2) - (4)^2 = 12x + 12 - 16 = 12x - 4$$

$$x = 0 \rightarrow \Delta = -4 < 0 \rightarrow \underline{\text{Saddle Point}}$$

$$x = \frac{2}{3} \rightarrow \Delta = 4 > 0 \rightarrow f_{xx} = 10 > 0 \rightarrow \text{min}$$

نقاط بحرانی تابع  $z = x^3 + xy + y^2 - 5x - 5y$  چگونه هستند؟

$$\begin{aligned} z_x &= 3x^2 + y - 5 = 0 \\ z_y &= x + 2y - 5 = 0 \end{aligned} \rightarrow \begin{aligned} -9x^2 - 2y + 10 &= 0 \\ x + 2y - 5 &= 0 \\ -9x^2 + x + 10 &= 0 \end{aligned} \rightarrow \begin{cases} x = 1 \\ x = -10/9 \end{cases}$$

$$\begin{aligned} z_{xx} &= 6x & z_{yy} &= 2 & z_{xy} &= 1 \\ \Delta &= (6x)(2) - 1 = 12x - 1 \end{aligned}$$

$$\begin{aligned} x = 1 &\rightarrow \Delta > 0 \rightarrow z_{xx} > 0 \rightarrow \underline{\underline{\min}} \\ x = -10/9 &\rightarrow \Delta < 0 \rightarrow \text{نقطه سرجی} \end{aligned}$$

$$f = 9x^2 + 4y^2 + z^2 \quad \underline{\underline{9x + 4y + z = 14}}$$

$$\frac{18x}{9} = \frac{8y}{4} = \frac{2z}{1} \rightarrow 2x = 2y = z$$

$$9x + 4x + 2x = 14 \rightarrow 14x = 14 \rightarrow x = 1 \quad \checkmark$$

$$\begin{array}{l} x=1 \\ y=1 \\ z=2 \end{array} \rightarrow f = 9 + 4 + 4$$

$$Z = x^{\nu} + y^{\mu}$$

$$\underline{\underline{\nu x + \mu y = \nu \mu}}$$

$$\frac{\nu x}{\nu} = \frac{\mu y}{\mu} \rightarrow x = \frac{\mu}{\nu} y = \frac{\nu}{\mu} \times \frac{\nu \mu \times \nu}{\nu} = \frac{\nu \mu \times \nu}{\nu}$$

$$\nu \left( \frac{\mu}{\nu} y \right) + \mu y = \nu \mu \rightarrow \frac{\nu}{\nu} \mu y + \mu y = \frac{\nu \mu}{\nu} = \nu \mu \rightarrow y = \frac{\nu \mu \times \nu}{\nu}$$

$$Z = x^2 + y^2 + 4xy \quad x + y = 11$$

$$\frac{1x + 4y}{1} = \frac{4y + 1x}{1} \rightarrow y = x \rightarrow y = 11 - x$$

$$y = 11 - x \rightarrow x + 11 - x = 11 \rightarrow \underline{x = 0} \rightarrow \underline{y = 11}$$

$$Z = 0^2 + 11^2 + 4 \cdot 0 \cdot 11 = 121$$

کمترین مقدار تابع  $u = x^2 + y^2 + z^2$  با شرط  $2x - y + 2z = 6$  کدام است؟

$$\frac{2x}{2} = \frac{-y}{-1} = \frac{2z}{2} \rightarrow x = z = -y \quad \checkmark$$

$$2x + \frac{x}{2} + 2x = 6 \rightarrow \frac{9x}{2} = 6 \rightarrow x = \frac{12}{9} = \frac{4}{3} \quad \checkmark$$

$$x = \frac{4}{3} \rightarrow z = \frac{4}{3} \rightarrow y = -\frac{4}{3}$$

$$u = \frac{16}{9} + \frac{16}{9} + \frac{16}{9} = \frac{48}{9} = 16 \quad \checkmark$$



بیشترین مقدار عبارت  $u = xyz$  با شرط  $4x + 2y + z = 2$  کدام است؟

$$\frac{yz}{4} = \frac{xz}{2} = \frac{xy}{1}$$

$$\frac{y}{4} = \frac{z}{2} \rightarrow y = 2z$$

$$\frac{z}{2} = \frac{y}{1} \rightarrow z = 2y \rightarrow z = 4x$$

$$4x + 4x + 4x = 2 \rightarrow x = \frac{1}{6} \rightarrow y = \frac{1}{3} \rightarrow z = \frac{2}{3}$$

$$U = xyz = \frac{1}{6} \times \frac{1}{3} \times \frac{2}{3} = \frac{1}{9}$$

بیشترین مقدار تابع  $W = x^2yz$  با شرط  $2x + y + 2z = 24$ ، کدام است؟

$$\frac{2xyz}{2} = \frac{x^2z}{1} = \frac{x^2y}{2} \rightarrow \left| \begin{array}{l} \frac{z}{1} = \frac{y}{2} \rightarrow y = 2z \\ x = y \rightarrow x = 2z \end{array} \right.$$

$$2(2z) + 2z + 2z = 24 \rightarrow 12z = 24 \rightarrow z = 2 \rightarrow \left| \begin{array}{l} y = 4 \\ x = 4 \end{array} \right.$$

$$W = x^2yz = 4^2 \times 4 \times 2$$

$$\Delta > 0 \rightarrow \begin{cases} Z_{xx} > 0 & \text{محدب} \\ Z_{xx} < 0 & \text{مقعر} \end{cases}$$

$$Z = 1x^2 + 1y^2 - xy - 1x - 1y$$

$$Z_x = 2x - y - 1 \rightarrow Z_{xx} = 2, \quad Z_{xy} = -1$$

$$Z_y = 2y - x - 1 \rightarrow Z_{yy} = 2, \quad Z_{xy} = -1$$

$$\Delta = Z_{xx} \cdot Z_{yy} - (Z_{xy})^2 = 2 \cdot 2 - (-1)^2 = 3 > 0$$

$$\Delta > 0 \rightarrow Z_{xx} = 2 > 0$$

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