

# آموزش ریاضی

## زاویه های خاص

### علی هاشمی

کلیه حقوق مادی و معنوی این اثر متعلق به سایت خانه ریاضی علی هاشمی است و هرگونه استفاده از این اثر و انتشار آن در پایگاه های مجازی بدون کسب مجوز ممنوع است و متخلفان تحت پیگرد قانونی قرار می گیرند.

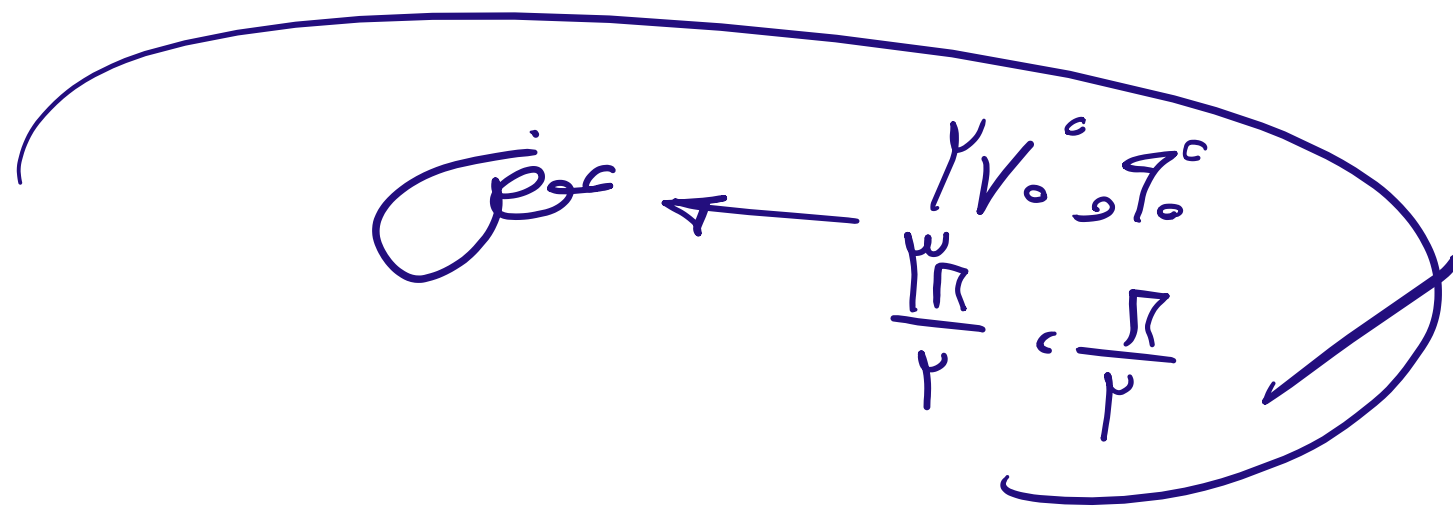
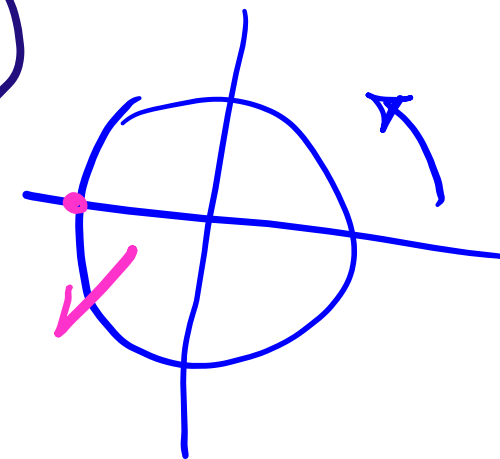
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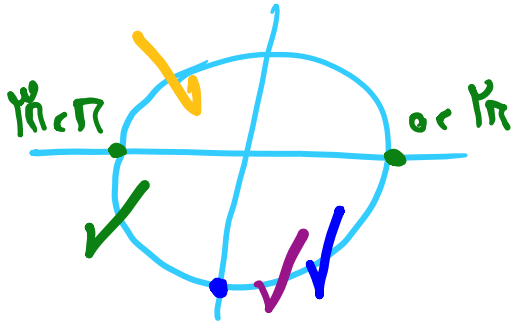
$$\begin{aligned} \sin 110^\circ &\rightarrow \left\{ \begin{aligned} \sin(90^\circ + 20^\circ) &= +\cos 20^\circ = \frac{\sqrt{3}}{2} \\ \sin(110^\circ - 20^\circ) &= +\sin 20^\circ = \frac{\sqrt{3}}{2} \end{aligned} \right. \end{aligned}$$

۹۰ و ۲۷۰ عبارت مستثنی از تغییر می دهند.



$$\begin{aligned} \sin \frac{10\pi}{3} &= \sin \left( \frac{9\pi}{3} + \frac{\pi}{3} \right) = \sin \left( \frac{3\pi}{1} + \frac{\pi}{3} \right) \\ &= -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2} \end{aligned}$$





کدام است؟  $\frac{\sin(\alpha - \frac{\pi}{2}) + \sin(\frac{3\pi}{2} + \alpha)}{\cos(\frac{3\pi}{2} + \alpha) + \cos(\alpha - \pi)}$  اگر  $\tan \alpha = \frac{2}{3}$  باشد مقدار ۱

$$\frac{-\cos \alpha - \sin \alpha}{\sin \alpha - \cos \alpha}$$

$\div \cos \alpha$

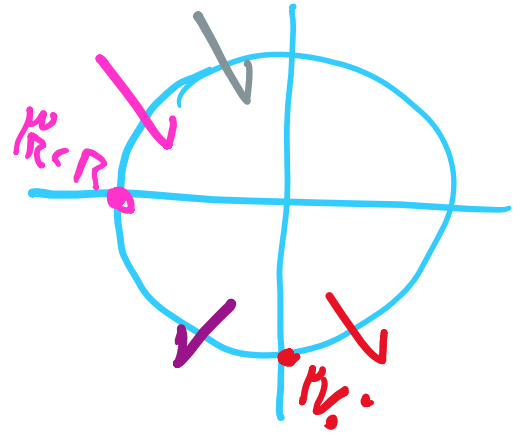
$$\frac{-1 - \tan \alpha}{\tan \alpha - 1} = \frac{-1 - \frac{2}{3}}{\frac{2}{3} - 1}$$

$$\frac{-\frac{5}{3}}{-\frac{1}{3}} = +5$$

$\sin(-a) = -\sin a$        $\sin(a - \frac{\pi}{2}) = -\sin(\frac{\pi}{2} - a)$   
 $\cos(-a) = \cos a$        $\cos(a - \pi) = \cos(\pi - a)$

حاصل عبارت  $\frac{\cos 285^\circ - \sin 255^\circ}{\sin 525^\circ - \sin 105^\circ}$ ، با فرض  $\tan 15^\circ = 0,28$  کدام است؟ ۲

$$\frac{\cos(270+15) - \sin(270-15)}{\sin(540-15) - \sin(90+15)} = \frac{+\sin 15 + \cos 15}{+\sin 15 - \cos 15}$$



$$\xrightarrow{\div \cos 15} \frac{\tan 15 + 1}{\tan 15 - 1} = \frac{0,28 + 1}{0,28 - 1} = \frac{1,28}{-0,72}$$

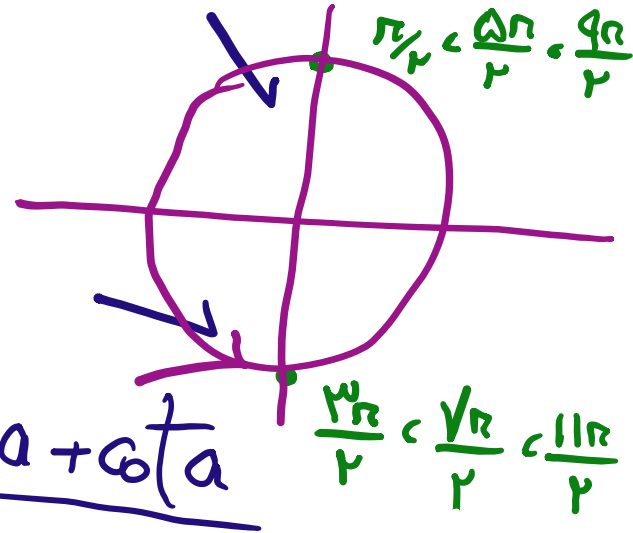
$$\rightarrow -\frac{128}{72} = -\frac{16}{9}$$



$$\boxed{\cot a = \frac{3}{4}}$$

اگر  $\tan \alpha = \frac{4}{3}$  و انتهای کمان  $\alpha$  در ربع سوم باشد، حاصل عبارت زیر کدام است؟

$$\sin\left(\frac{9\pi}{2} + \alpha\right) \cos\left(\frac{7\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$$



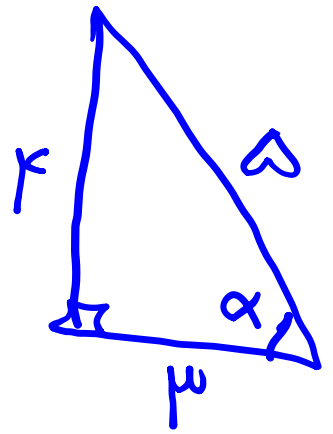
$$= +\cos a \cdot (-\sin a) + \cot a = -\sin a \cos a + \cot a$$

$$1 + \tan^2 a = \frac{1}{\cos^2 a} \rightarrow \cos^2 a = \frac{9}{25} \rightarrow \cos a = -\frac{3}{5}$$

$$1 + \cot^2 a = \frac{1}{\sin^2 a} \rightarrow \sin^2 a = \frac{16}{25} \rightarrow \sin a = -\frac{4}{5}$$

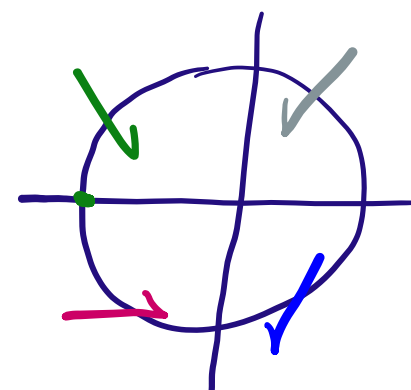
$$\therefore \text{جواب} = -\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4} = \frac{27}{100}$$

$$-\tan\left(a - \frac{3\pi}{2}\right) = +\tan\left(\frac{3\pi}{2} - a\right)$$



۴ اگر  $\tan 25^\circ = 0,48$  باشد حاصل عبارت  $\frac{\sin 155^\circ - 3 \cos 245^\circ}{\cos 295^\circ - 2 \sin 65^\circ}$  کدام است؟

$$\frac{\sin(180-25) - 3 \cos(180-25)}{\cos(180+25) - 2 \sin(90-25)} = \frac{+\sin 25 + 3 \sin 25}{+\sin 25 - 2 \cos 25}$$



$$\xrightarrow{\div \cos 25} \frac{\tan 25 + 3 \tan 25}{\tan 25 - 2} = \frac{4(0,48)}{0,48 - 2} = \frac{1,92}{-1,52}$$

$$\rightarrow -\frac{192}{152} = -\frac{24}{19}$$

حاصل عبارت  $\sin\left(\frac{17\pi}{3}\right) \cos\left(\frac{17\pi}{6}\right) + \tan\left(\frac{19\pi}{4}\right) \sin\left(\frac{-11\pi}{6}\right)$  کدام است؟ (5)

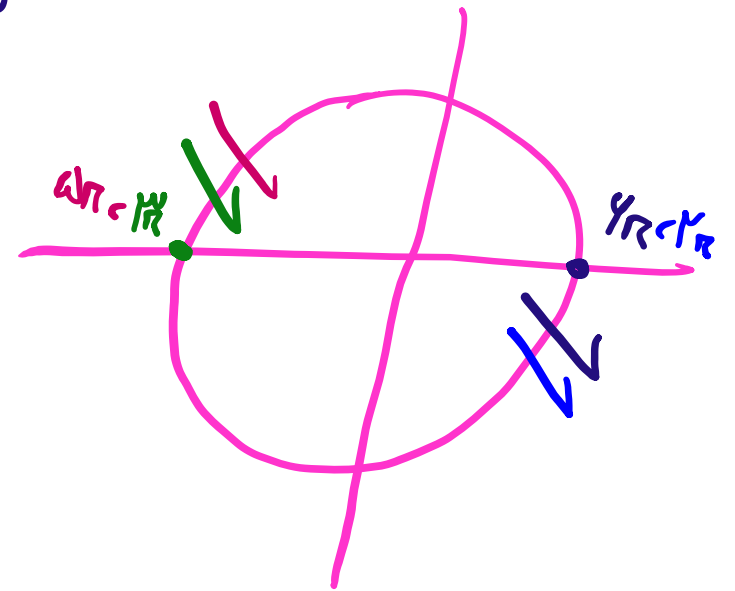
$$\sin\left(\frac{17\pi}{3} - \frac{\pi}{3}\right) = \sin\left(4\pi - \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{17\pi}{4} - \frac{\pi}{4}\right) = \cos\left(4\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\left(\frac{19\pi}{4} - \frac{\pi}{4}\right) = \tan\left(5\pi - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

$$\sin\left(\frac{11\pi}{4} - \frac{\pi}{4}\right) = \sin\left(3\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

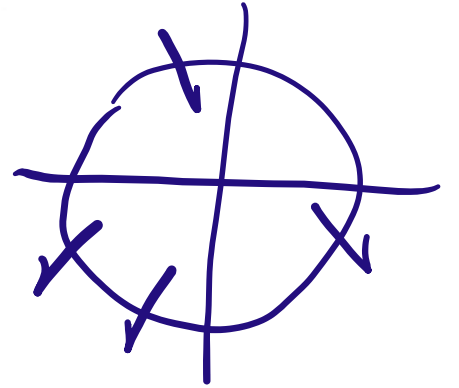
$$\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - (-1)\left(-\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{6}}{4} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$





حاصل عبارت  $\frac{\sin 25^\circ + \sin 70^\circ}{\cos 56^\circ - \cos 11^\circ}$  با فرض  $\tan 20^\circ = 0,4$  ، کدام است؟ ۶

$$\frac{\sin(20^\circ - 20^\circ) + \sin(70^\circ - 20^\circ)}{\cos(20^\circ + 20^\circ) - \cos(90^\circ + 20^\circ)} = \frac{-\cos 20^\circ - \sin 20^\circ}{-\cos 20^\circ + \sin 20^\circ}$$



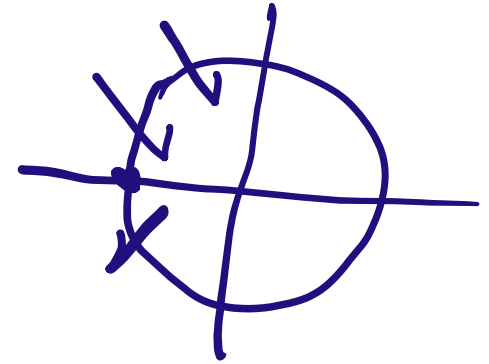
$$\div \cos 20^\circ \quad \frac{-1 - \tan 20^\circ}{-1 + \tan 20^\circ} = \frac{-1 - 0,4}{-1 + 0,4} = \frac{-1,4}{-0,4}$$

$$\rightarrow \frac{1,4}{0,4} = \frac{7}{2}$$

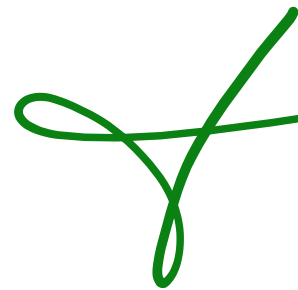


خلاصه شده‌ی عبارت  $\sin\left(\frac{\pi}{2} + \alpha\right) \sin(\pi + \alpha) - \sin(\pi - \alpha) \cos(\alpha)$  کدام است؟ (۷)

$$(+\cos\alpha)(-\sin\alpha) - (\sin\alpha)(\cos\alpha)$$



$$= -\sin\alpha \cdot \cos\alpha - \sin\alpha \cdot \cos\alpha = -2\sin\alpha \cdot \cos\alpha$$



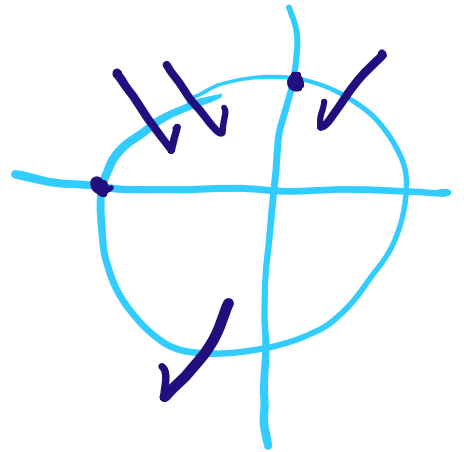
اگر  $\cot 20^\circ = \frac{8}{3}$  باشد حاصل  $\frac{2 \sin 250^\circ - \cos 160^\circ}{\sin 160^\circ + 3 \cos 70^\circ - \sin 110^\circ}$  برابر کدام است؟ ۸

$$\frac{2 \sin(270^\circ - 20^\circ) - \cos(180^\circ - 20^\circ)}{\sin(180^\circ - 20^\circ) + 3 \cos(90^\circ - 20^\circ) - \sin(90^\circ + 20^\circ)} = \frac{-2 \cos 20^\circ + \cos 20^\circ}{\sin 20^\circ + 3 \sin 20^\circ - \cos 20^\circ}$$

$$= \frac{-\cos 20^\circ}{4 \sin 20^\circ - \cos 20^\circ}$$

$\div \sin 20^\circ$

$$\frac{-\cot 20^\circ}{4 - \cot 20^\circ}$$



$$= \frac{-\frac{1}{\mu}}{4 - \frac{1}{\mu}} = \frac{-\frac{1}{\mu}}{\frac{4\mu - 1}{\mu}} = -\frac{1}{4\mu - 1}$$



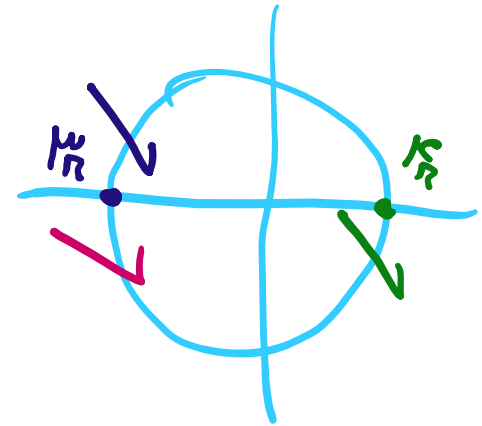
۹ حاصل عبارت  $\tan \frac{11\pi}{4} + \sin \frac{15\pi}{4} \cos \frac{13\pi}{4}$  کدام است؟

$$\tan\left(\frac{11\pi}{4} - \frac{\pi}{4}\right) = \tan\left(\frac{10\pi}{4} - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

$$\sin\left(\frac{14\pi}{4} - \frac{\pi}{4}\right) = \sin\left(\frac{13\pi}{4} - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{12\pi}{4} + \frac{\pi}{4}\right) = \cos\left(\frac{13\pi}{4} + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$-1 + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -1 + \frac{1}{2} = -\frac{1}{2}$$



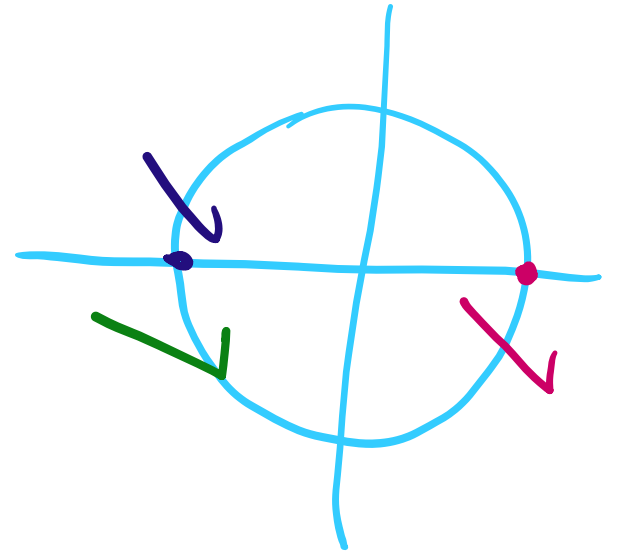
حاصل عبارت  $\tan \frac{17\pi}{6} \sin \frac{11\pi}{3} + \cos \frac{10\pi}{3}$  کدام است؟ (۱۰)

$$\tan\left(\frac{11\pi}{4} - \frac{\pi}{4}\right) = \tan\left(\frac{10\pi}{4}\right) = -\tan \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{11\pi}{6} - \frac{\pi}{6}\right) = \sin\left(\frac{10\pi}{6}\right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{9\pi}{6} + \frac{\pi}{6}\right) = \cos\left(\frac{10\pi}{6}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\text{جواب} = \left(-\frac{\sqrt{2}}{2}\right) \left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$$



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