

آموزش ریاضی

توان و ریشه

علی هاشمی

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^m \cdot b^m = (ab)^m$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$x^2 \cdot x^3 = x^5$$

$$\frac{x^5}{x^3} = x^2 = 9$$

$$5^2 \cdot 2^2 = 10^2 = 100$$

$$\frac{100^5}{10^5} = 10^5$$

فرمول های ضرب و تقسیم

$$(a^m)^n = a^{mn}$$

$$(x^2)^5 = x^{10}$$

$$\underline{\underline{x^2 \cdot x^5 = x^7}}$$



$$a^{-m} = \frac{1}{a^m}$$

$$r^{-5} = \frac{1}{r^5} = \frac{1}{r^5}$$

$$\left(\frac{r}{r}\right)^{-3} = \left(\frac{r}{r}\right)^{+3} = \frac{1}{r^3}$$

$$\frac{r}{r^{-4}} = r^1 \times r^4 = r^5 = r^5$$



$$\textcircled{1} a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \mu^{\frac{\omega}{\nu}} = \sqrt[\nu]{\mu^\omega}$$

$$\sqrt[\nu]{x^\omega} \times \sqrt[\mu]{x'} = x^{\frac{\omega}{\nu}} \cdot x^{\frac{1}{\mu}} = x^{\frac{\omega + \nu}{\nu}} = x^{\frac{17}{4}} = \sqrt[4]{x^{17}}$$

$$1^{-\frac{\nu}{\mu}} = \frac{1}{1^{\frac{\nu}{\mu}}} = \frac{1}{\sqrt[\mu]{1^\nu}} = \frac{1}{\mu^\nu} = \frac{1}{\mu^\nu}$$

$$\textcircled{2} a \sqrt[n]{b} = \sqrt[n]{a^n \cdot b}$$

$$\textcircled{3} \sqrt[\mu]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\nu \sqrt[\mu]{a} = \sqrt[\mu]{\nu^\mu \times a} = \sqrt[\mu]{\nu^\mu a}$$

$$\sqrt[\nu]{\sqrt[\omega]{x}} = \sqrt[\nu\omega]{x}$$



گویا کردن مخرج کسر

$$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$\frac{5}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{5(\sqrt{2}+1)}{2-1} = 5(\sqrt{2}+1)$$



محاسبه رادیکال با قدر مطلق

$$\sqrt{(-3)^2} = |-3| = +3 \quad \checkmark$$

$$\sqrt{(-5)^2} = |-5| = 5 \quad \checkmark$$

$$\sqrt{(\sqrt{2}-1)^2} = |\sqrt{2}-1| = \sqrt{2}-1 \quad \checkmark$$

$$\sqrt{(1-\sqrt{3})^2} = |1-\sqrt{3}| = -1+\sqrt{3} \quad \checkmark$$

$$\sqrt{(-3)^2} = 3$$

$$\sqrt{x^2} = x$$

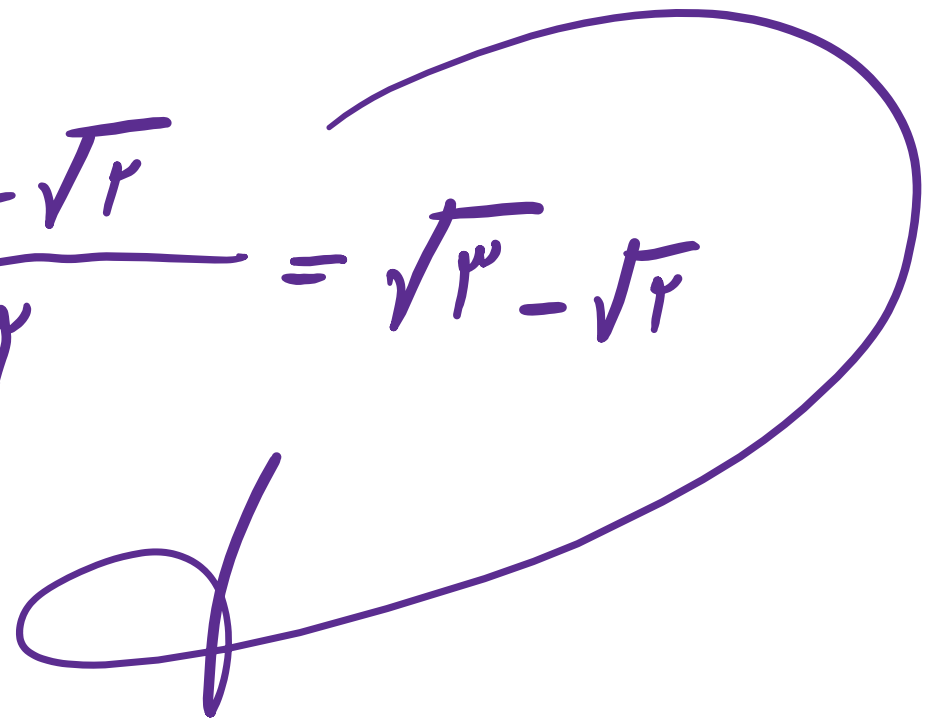
$$\sqrt{x^2} = |x|$$



۱ حاصل کسر $\frac{1}{-\sqrt[3]{8} - \sqrt{50} + \sqrt{3}}$ کدام است؟

$$\frac{1}{4\sqrt{2} - 5\sqrt{2} + \sqrt{10}} = \frac{1}{\sqrt{2} + \sqrt{10}}$$

$$\frac{1}{\sqrt{10} + \sqrt{2}} \times \frac{\sqrt{10} - \sqrt{2}}{\sqrt{10} - \sqrt{2}} = \frac{\sqrt{10} - \sqrt{2}}{10 - 2} = \sqrt{10} - \sqrt{2}$$



$$(a^m)^n = a^{mn}$$

کدام است؟

$$24^{\frac{2}{3}} \times 4^{\frac{7}{2}}$$

ریشه بیست و سوم عبارت

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$$24^{\frac{2}{3}} = \left(2 \times 3 \right)^{\frac{2}{3}} = 2^{\frac{2}{3}} \times 3^{\frac{2}{3}}$$

$$4^{\frac{7}{2}} = \left(2^2 \right)^{\frac{7}{2}} = 2^7$$

$$24^{-\frac{1}{4}} = \left(2 \times 3 \right)^{-\frac{1}{4}} = 2^{-\frac{1}{4}} \times 3^{-\frac{1}{4}}$$

$$32^2 \times 27^{-\frac{1}{3}} \times 48^{-\frac{1}{4}}$$

$$32^2 = (2^5)^2 = 2^{10}$$

$$(27)^{-\frac{1}{3}} = (3^3)^{-\frac{1}{3}} = 3^{-1}$$

$$\rightarrow A = \frac{2^2 \times 3^{\frac{2}{3}} \times 2^7}{2^{10} \times 3^{-1} \times 2^{-\frac{1}{4}} \times 3^{-1}}$$

$$= \frac{2^9 \times 3^{\frac{2}{3}}}{2^9 \times 3^{-2}} = 3^{\frac{2}{3} + 2} = 3^{\frac{14}{3}}$$

$$\sqrt[14]{A} = A^{\frac{1}{14}} = \left(3^{\frac{14}{3}} \right)^{\frac{1}{14}} = 3^{\frac{1}{3}} = \sqrt[3]{3}$$



حاصل عبارت $A = \frac{2}{2 + \sqrt{3}} + \sqrt{13 + 4\sqrt{3}}$ کدام است؟ (۳)

$$\frac{2}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{4 - 2\sqrt{3}}{4 - 3} = 4 - 2\sqrt{3}$$

$$\sqrt{13 + 4\sqrt{3}} = \sqrt{1 + 2(2\sqrt{3}) + 12} = \sqrt{(1 + 2\sqrt{3})^2} = 1 + 2\sqrt{3}$$

$a^2 + 2(ab) + b^2$

$$\rightarrow 4 - 2\sqrt{3} + 1 + 2\sqrt{3} = 5$$



حاصل $\sqrt[5]{(\sqrt{2}+1)^4} \times \sqrt[5]{(3-2\sqrt{2})^2}$ کدام است؟

$$\sqrt[5]{((\sqrt{2}+1)^2)^2} = \sqrt[5]{(2+1+2\sqrt{2})^2} = \sqrt[5]{(3+2\sqrt{2})^2}$$

$$\rightarrow (3+2\sqrt{2})^{\frac{2}{5}} \times (3-2\sqrt{2})^{\frac{2}{5}} = (9-1)^{\frac{2}{5}} = 1$$

$$\sqrt[5]{(\sqrt{2}+1)^4} \times \sqrt[5]{(\sqrt{2}-1)^4} = 1$$



ساده شده‌ی عبارت

کدام است؟ $\left(\sqrt[3]{5 + \left(-\frac{1}{\sqrt{3}}\right)^3 + (-\sqrt{3})^3} \right)^{-\frac{1}{3}} + \left(\sqrt[3]{\frac{2}{3} - \frac{1}{3}} \right)^3$

$$\left(\sqrt[3]{5 + \frac{1}{9} + 3} \right)^{-\frac{1}{3}} = \left(\sqrt[3]{\frac{44}{9}} \right)^{-\frac{1}{3}} = \left(\frac{2^2 \cdot 11}{3^2} \right)^{-\frac{1}{3}} = \frac{3^{-2}}{2^{-2} \cdot 11^{-1}} = \frac{3^2}{2^2 \cdot 11}$$

$$\left(\sqrt{\frac{9}{4}} \right)^{\frac{3}{2}} = \left(\frac{3}{2} \right)^{\frac{3}{2}} = \frac{3^{\frac{3}{2}}}{2^{\frac{3}{2}}} = \frac{3\sqrt{3}}{2\sqrt{2}}$$

$$\rightarrow \frac{3}{1} + \frac{2\sqrt{2}}{1} = \frac{3 + 2\sqrt{2}}{1} = \frac{3}{1} + \frac{2\sqrt{2}}{1}$$



کدام است؟ حاصل $\frac{2}{\sqrt{5}-1} \div \left(\frac{2\sqrt{5}-2}{4}\right)^2$

$$\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{5-1} = \frac{\sqrt{5}+1}{2}$$

$$\left(\frac{2(\sqrt{5}-1)}{4}\right)^2 = \frac{5+1-2\sqrt{5}}{4} = \frac{4-2\sqrt{5}}{4} = \frac{2-\sqrt{5}}{2}$$

$$\frac{\sqrt{5}+1}{2} \times \frac{2}{2-\sqrt{5}} = \frac{\sqrt{5}+1}{2-\sqrt{5}}$$

$$\frac{\sqrt{5}+1}{2} + \frac{2-\sqrt{5}}{2} = 2$$



حاصل عبارت $\frac{1-\sqrt{2}}{1+\sqrt{2}} - \frac{4\sqrt{6}}{\sqrt{12}}$ کدام است؟ ۷

$$\frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{(1-\sqrt{2})^2}{1-2} = - \left(\frac{1+2-2\sqrt{2}}{1} \right) = -1^3 + 2\sqrt{2}$$

$$\frac{4\sqrt{6}}{\sqrt{12}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$\rightarrow -1^3 + \cancel{2\sqrt{2}} - \cancel{2\sqrt{2}} = -1$$



اگر $A = 2\sqrt[2]{50} + 4\sqrt[3]{75} - 5\sqrt[14]{48} - 3\sqrt[4]{8}$ باشد، A^2 برابر کدام است؟ ۸

$$A = 10\sqrt{2} + 20\sqrt{3} - 10\sqrt{3} - 4\sqrt{2}$$

$$A = 4\sqrt{2}$$

$$\rightarrow A^2 = 16 \times 2 = 32$$



۹ حاصل عبارت $\sqrt[3]{24} \times \sqrt[3]{9} + \frac{2-\sqrt{5}}{2+\sqrt{5}} - \sqrt{80}$ کدام است؟

$$2\sqrt[3]{3} \times \sqrt[3]{9} = 2\sqrt[3]{27} = 2 \times 3 = 6$$

$$\frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{(2-\sqrt{5})^2}{4-5} = -\left(4+5-4\sqrt{5}\right) = -9+4\sqrt{5}$$

$$\sqrt{16 \times 5} = 4\sqrt{5}$$

$$6 - 9 + 4\sqrt{5} - 4\sqrt{5} = -3$$



۱۰ حاصل عبارت $\sqrt[3]{25} \times \sqrt[3]{40} + \frac{\sqrt{2}}{3+2\sqrt{2}} - \frac{1}{2}\sqrt{72}$ کدام است؟

$$\sqrt[3]{5^2} \times \sqrt[3]{2^3 \times 5} = \sqrt[3]{2^3 \times 5^3} = 2 \times 5 = 10 \quad \checkmark$$

$$\frac{\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3\sqrt{2}-4}{9-8} = 3\sqrt{2}-4 \quad \checkmark$$

$$\frac{1}{2}\sqrt{72} = \frac{1}{2}\sqrt{4 \times 2 \times 9} = \sqrt{2} \quad \checkmark$$

$$\rightarrow 10 + \cancel{3\sqrt{2}-4} - \cancel{\sqrt{2}} = 9 \quad \checkmark$$



حاصل $\frac{\sqrt{3}}{3} \times \sqrt{48} + \sqrt{20} - \frac{2}{2+\sqrt{5}}$ ، کدام است؟ (11)

$$\frac{2}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{4-2\sqrt{5}}{4-5} = -4+2\sqrt{5}$$

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

$$\sqrt{48} \times \frac{\sqrt{3}}{3} = \sqrt{16 \times 3} \times \frac{\sqrt{3}}{3} = 4 \times 1 = 4$$

$$\rightarrow -4+2\sqrt{5} - 2\sqrt{5} + 4 = 0$$



۱۲) اگر $A = 0,2\sqrt[3]{125} + \sqrt[3]{56}$ باشد، حاصل $A^3 - 3A^2 + 3A$ کدام است؟

$$A^3 - 3A^2 + 3A - 1 + 1 = (A-1)^3 + 1$$

$$A = 0,2\sqrt[3]{5^3} + \sqrt[3]{2^3 \times 7} = 1 + 2\sqrt[3]{7}$$

$$\rightarrow (1 + 2\sqrt[3]{7} - 1)^3 + 1 = 8 \times 7 + 1 = 57$$



حاصل عبارت $(\sqrt[3]{2})(\sqrt[3]{2-\sqrt{3}})$ کدام است؟

$$\sqrt[3]{(\sqrt{3}+1)^2} \cdot \sqrt[3]{(\sqrt{3}-1)^2}$$

$$= \sqrt[3]{(\sqrt{3}-1)^2} = \sqrt[3]{2^2} = 2^{\frac{2}{3}}$$

$$\sqrt[3]{2-2\sqrt{3}} = \sqrt[3]{\sqrt{3}+1-2\sqrt{3}} = \sqrt[3]{(\sqrt{3}-1)^2} \neq \sqrt[3]{2}$$



۱۴ اگر $x = 5 + \sqrt{17}$ باشد، حاصل عبارت $\sqrt{\frac{x-1}{16} + \frac{1}{2x}}$ ، کدام است؟

$$\sqrt{\frac{x^2 - x + 1}{16x}} = \frac{1}{4} \sqrt{x - 1 + \frac{1}{x}} = \frac{1}{4} \sqrt{5 + \sqrt{17} - 1 + \frac{1}{5 + \sqrt{17}}}$$

$$= \frac{1}{4} \sqrt{5 + \sqrt{17} - 1 + 5 - \sqrt{17}} = \frac{1}{4} \sqrt{9} = \frac{3}{4} = 0.75$$

$$\frac{1}{5 + \sqrt{17}} \times \frac{5 - \sqrt{17}}{5 - \sqrt{17}} = \frac{1(5 - \sqrt{17})}{25 - 17} = 5 - \sqrt{17}$$



۱۵ اگر $A = \frac{2}{3}\sqrt{18} + 2\sqrt{27} - \sqrt{108} + 0,3\sqrt{200}$ باشد، A^2 برابر کدام است؟

$$\frac{2}{3}\sqrt{9 \times 2} = 2\sqrt{2}$$

$$2\sqrt{9 \times 3} = 6\sqrt{3}$$

$$\sqrt{108} = \sqrt{9 \times 3 \times 3 \times 3} = 6\sqrt{3}$$

$$0,3\sqrt{100 \times 2} = \frac{3}{10}\sqrt{2}$$

$$\rightarrow 2\sqrt{2} + 6\sqrt{3} - 6\sqrt{3} + \frac{3}{10}\sqrt{2} = \frac{23}{10}\sqrt{2}$$

$$A^2 = \left(\frac{23}{10}\sqrt{2}\right)^2 = \frac{23^2 \times 2}{100} = \frac{1058}{50} = 21,16$$



حاصل $(2\sqrt{3} - 3\sqrt{2})(\sqrt{3} + \sqrt{2})$ کدوم است؟

$$\frac{2}{2 + \sqrt{4}} \times \frac{2 - \sqrt{4}}{2 - \sqrt{4}} = \frac{2(2 - \sqrt{4})}{4 - 4} = -2 + \sqrt{4}$$

$$\rightarrow 4 + 2\sqrt{4} - 3\sqrt{4} - 4 = -\sqrt{4}$$

$$\rightarrow -2 + \sqrt{4} - \sqrt{4} = -2$$

$$\begin{aligned} \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} &= 2 \\ \frac{-2}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} &= -2 \end{aligned}$$



اگر $A = \sqrt[3]{\frac{x-1}{36} - \frac{1}{x} + \frac{\sqrt{2}}{36}}$ باشد، با فرض $x = 1 - \sqrt{2}$ حاصل A^3 کدام است؟ (۱۷)

$$A^3 = \frac{x-1}{36} - \frac{1}{x} + \frac{\sqrt{2}}{36} = \frac{-\sqrt{2}}{36} - \frac{1}{1-\sqrt{2}} + \frac{\sqrt{2}}{36}$$

$$= 1 + \sqrt{2}$$

$$\frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2} = -(1+\sqrt{2})$$



مقدار $\sqrt{7-4\sqrt{3}} - 2\sqrt{7+4\sqrt{3}} + \sqrt{27}$ کدوم است؟ $9\sqrt{3}$

$$\sqrt{4+3-4\sqrt{3}} = \sqrt{(2-\sqrt{3})^2} = |2-\sqrt{3}| = 2-\sqrt{3}$$

$$\sqrt{4+3+4\sqrt{3}} = \sqrt{(2+\sqrt{3})^2} = |2+\sqrt{3}| = 2+\sqrt{3}$$

$$\rightarrow \underline{2-\sqrt{3}} - \underline{4-2\sqrt{3}} + \underline{3\sqrt{3}} = -2$$



حاصل عبارت $\frac{\sqrt{8} + \sqrt{6}}{\sqrt{2}} - \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ کدام است؟

$$\frac{\sqrt{8} + \sqrt{6}}{\sqrt{2}} = \frac{\sqrt{8}}{\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{4} + \sqrt{3} = \underline{2 + \sqrt{3}}$$

$$\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{(1 - \sqrt{3})^2}{1 - 3} = \frac{1 + 3 - 2\sqrt{3}}{-2} = \frac{4 - 2\sqrt{3}}{-2}$$

$$\rightarrow \frac{4}{-2} - \frac{2\sqrt{3}}{-2} = \underline{-2 + \sqrt{3}}$$

$$\rightarrow 2 + \sqrt{3} + (-2 + \sqrt{3}) = \underline{2\sqrt{3}}$$



خلاصه شده‌ی عبارت $\sqrt[3]{\frac{3^3}{8}} \times (18)^{-2} \times (1,5)^4 \times \left(\frac{1}{4}\right)^{-3}$ کدام است؟

$$\sqrt[3]{\frac{3^3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^4$$

$$\left(\frac{1}{4}\right)^{-3} = 4^{3+3} = (2^2)^3 = 2^6$$

$$(18)^{-2} = \frac{1}{(3^2 \times 2^2)^2} = \frac{1}{3^4 \times 2^4}$$

$$\begin{aligned} \rightarrow \frac{3 \times 3^4 \times 2^6}{2 \times 3^4 \times 2^4 \times 2^4} &= \frac{3^5 \times 2^6}{3^4 \times 2^8} = \frac{3}{2} = 1,5 \end{aligned}$$



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